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MHD INSTABILITY OF ANISOTROPIC TWO-COMPONENT PLASMA. I. THE CASE WITHOUT HEAT FLUX

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On the basis of 16-momentum MHD equations we examined the propagation of linear waves in an anisotropic homogeneous cosmic plasma. We obtained general dispersion equation taking into account two components of plasma (electrons and protons) and heat flux along magnetic field. Obtained dispersion equation is a generalization of earlier studied cases, when plasma was ionic. The case when the effects related to heat flux are ignored was analyzed in details. Within the limit of longitudinal propagation we give a classification of wave modes which completely correspond to the known modes of kinetic physics of collisionless plasma. Fire-hose and mirror instabilities were analyzed. It is shown that the inclusion of electrons changes the instability criterion.

Keywords: Magnetohydrodynamics (MHD) – 16-momentum transport equation – Instabilities – Collisionless plasma

1. INTRODUCTION

In the cosmic plasma conditions the theory of wave turbulence has a special importance. As a rule, all observed significant dynamic phenomena occur on a large scale both on time and in space. If plasma is sufficiently dense and collisionally dominated, distribution functions of particles are close to Maxwell distribution and fluid description is successfully used for description of such phenomena. In this case plasma is isotropic and usual MHD equations are applicable. However, in many cases (for example, in the coronas of stars, astrophysical disks, in the winds of stars, in heliosphere and magnetospheres of planets, etc.) plasma is strongly rarefied and weakly collisional. In such cases, especially when the influence of external magnetic field should be taken into consideration, plasma

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becomes anisotropic and usual MHD equations are not applicable. In such situations transverse and longitudinal kinetic temperatures (that is a pressure) differ, and plasma becomes nonequilibrium. Besides, a heat flux is appeared along the magnetic field. All these effects are the sources of a plasma instability [15] which cannot occur in usual isotropic plasma. For example, classical fire-hose and mirror instabilities [2,10,18,21] arise as a result of pressure anisotropy. Having from the space experiments and observations physical characteristics of large-scale phenomena in weakly collisional plasma, and for the purpose of applying kinetic modeling of plasma it is necessary to restore the distribution functions of particles.

In seven-dimensional space (spatial coordinates, chaotic velocities and time) to solve such problem is very difficult. Therefore it is necessary to derive the transport equations for the moments of the distribution function. However, in a such approach there arises an infinite set of equations clousure of which demands additional conditions. For example, the closed system of equation can be derived, if to it is suppose that the distribution functions of particles of anisotropic nonequilibrium plasma are close to Bi-Maxwellian distributions [1]. C.G.L. equations are the simplest example of such approach [3]. However these equations don't consider a heat flux along magnetic field. More comprehensive ones are 16-moment MHD transport equations which consider a heat flux [17,19]. These transport equations were used for modeling of solar wind [4,13,14,16] and studying MHD-instabilities in anisotropic plasma [5,6,11,12]. However, in these papers one-component (ionic plasma) was examined. This simplifying approach is reached by the assumption of cold electrons,$T_e << T_i$. The role of electrons in this case is reduced only for providing quasi-neutrality of plasma. However, in realistic practice this approach can be met only under special conditions [15]. For example, the ionic and electronic temperatures measured in solar wind generally are of one order [20]. Besides, if the effects connected with a heat flux become important, the participation of electrons in heat-transfer is important and must be considered.

In this paper the results of [7–9] are generalized for the two-component plasma. Linear waves in homogeneous anisotropic and non-isothermal magnetoactive plasma were examined with consideration of a heat flux along magnetic field. The general dispersion equation which turns to the known results for cold electrons was derived. Various special cases when derived equation is simplified were studied. Dependence of instability increment of compressible modes on the ratio of temperatures of ions and electrons was calculated. New criteria of fire-hose and mirror instability with consideration of the role of electrons were obtained.

2. MHD-EQUATIONS OF ANISOTROPIC PLASMA

For kinetic description of dynamic phenomena in plasma consisting of electrons and ions, the evolutionary equations of distribution functions of each type of particles - Boltzmann-Vlasov equations - are usually used. If the influence of electromagnetic field is considered, the Maxwell equations are added to these equations. For the distribution function of α type particles in seven-dimensional space $f_\alpha(\mathbf{r},\mathbf{u};t)$ the Boltzmann-Vlasov equation is presented in the following way

$$\frac{\partial f_\alpha}{\partial t}+\mathbf{u}_\alpha\nabla f_\alpha+\frac{1}{m_\alpha}\Big\{\mathbf{F}_\alpha+e_\alpha\left(\mathbf{E}+\frac{1}{c}\left[\mathbf{u}_\alpha\mathbf{B}\right]\right)\Big\}\nabla_u f_\alpha=Q_\alpha\left(f_\alpha\right), \qquad (1)$$

where $\mathbf{E}$ and $\mathbf{B}$ are intensity of electrical and magnetic fields correspondingly $\mathbf{u}_\alpha, e_\alpha$ and $m_\alpha-$ individual velocity, electric charge and mass of α type particle, $\mathbf{F}_\alpha-$ external force, $Q_\alpha-$ is the integral describing collision between particles. Macroscopic parameters of plasma (density, pressure, flux velocity, heat flux, viscosity etc.) are determined as the integral moments of the distribution functions in three-dimensional space of microscopic velocities: $M^{(k)}\equiv M^{(k)}_{\alpha_1\cdots\alpha_k}(\mathbf{r},t)=\int\prod_{j=1}^{k}u_{\alpha_j}f\,d^3\mathbf{u}$, $\alpha_i=1,2,3$. For example, the density of particles equals to $n\equiv n(\mathbf{r},t)=\int f\,d^3\mathbf{u}$, for velocity of plasma we have $n\,\mathrm{v}(\mathbf{r},t)=\int\mathbf{u}f\,d^3\mathbf{u}$, for difference of pressures across and along magnetic field we obtain

$$p_\perp-p_\parallel=\int m\left(u_\perp^2-\frac{u_\parallel^2}{2}\right)f\,d^3\mathbf{u}.$$

Here $d^3\mathbf{u}=du_x\,du_y\,du_z$. The number of these integral moments can be arbitrarily large and they all are expressed through each other. Equations which describe these moments (transport equations) can also be infinite. Additional physically justified conditions to be broken chain of the equations are required. In case of the dense plasma, when distribution functions of particles of equilibrium collisional plasma is close to Maxwell, these chains of equations are easily interrupted. As a result, usual MHD - equations for isotropic plasma are derived. However, in case of rare collisions and in the presence of strong magnetic field distribution functions of particles aren't described by Maxwell distribution, and a difficulty in interruption of a chain of the moment equations for nonequilibrium plasma arises. In this case longitudinal and transverse components of microscopic velocity appear as two independent variables on which the distribution function,

$f(\mathbf{r},\mathbf{u},t) \to f\left(\mathbf{r},\mathbf{u}_{\parallel},\mathbf{u}_{\perp},t\right)$ depends. The solution of the kinetic equation is usually given in the form of expansion

$$f(\mathbf{r},\mathbf{u},t) = f_0(\mathbf{r},\mathbf{u},t)\sum_{\nu} a_{\nu}(\mathbf{r},t)\,P_{\nu}(\mathbf{r},\mathbf{u},t)\,, \tag{2}$$

where f_0 is a zero approximation for distribution function, a_{ν} - are the expansion coefficients, P_{ν} - orthogonal polynomial with order ν. If we replace the function f_0 with bi-Maxwell function (the simplest form for anisotropic plasma), then

$$f_0 = n\left(\frac{m}{2\pi k_B T_{\perp}}\right)\left(\frac{m}{2\pi k_B T_{\parallel}}\right)^{1/2}\exp\left(-\frac{mw_{\perp}^2}{2k_B T_{\perp}} - \frac{mw_{\parallel}^2}{2k_B T_{\parallel}}\right), \tag{3}$$

where w=u-v .Using (3) in (2) and integrating the equation (1), we may derive chains of transport equations for the moments $M^{(k)}$. Then at very small Larmor radii of the rotation of particles around lines of force of magnetic field ($r_B \to 0$) we obtain the following 16-moment equations ([17, 19])

$$\frac{d\rho_{\alpha}}{dt} + \rho_{\alpha} div \mathrm{v} = 0, \tag{4}$$

$$\rho\frac{d\mathrm{v}}{dt} + \frac{1}{4\pi}\mathbf{B}\times(\nabla\times\mathbf{B}) + \sum_{\alpha}\left[\nabla p_{\alpha\perp} + (\mathbf{B}\cdot\nabla)\left(\frac{p_{\alpha\parallel}-p_{\alpha\perp}}{B^2}\mathbf{B}\right)\right] = 0, \tag{5}$$

$$Bp_{\alpha\parallel}\frac{d}{dt}\ln\left(\frac{B^2 p_{\alpha\parallel}}{n^3}\right) + \mathbf{B}\cdot\nabla S_{\alpha\parallel} + 2\left(S_{\alpha\perp} - \frac{1}{2}S_{\alpha\parallel}\right)\mathbf{B}\cdot\nabla\ln B = 0, \tag{6}$$

$$Bp_{\alpha\perp}\frac{d}{dt}\ln\left(\frac{p_{\alpha\perp}}{Bn}\right) + \mathbf{B}\cdot\nabla S_{\alpha\perp} - 2S_{\alpha\perp}\mathbf{B}\cdot\nabla\ln B = 0, \tag{7}$$

$$BS_{\alpha\parallel}\frac{d}{dt}\ln\left(\frac{B^3 S_{\alpha\parallel}}{2n^4}\right) + \frac{3p_{\alpha\parallel}}{m_{\alpha}}\mathbf{B}\cdot\nabla\left(\frac{p_{\alpha\parallel}}{n}\right) = 0, \tag{8}$$

$$BS_{\alpha\perp}\frac{d}{dt}\ln\left(\frac{S_{\alpha\perp}}{n^2}\right) + \frac{p_{\alpha\parallel}}{m_{\alpha}}\mathbf{B}\cdot\nabla\left(\frac{p_{\alpha\perp}}{n}\right) - \frac{\left(p_{\alpha\parallel}-p_{\alpha\perp}\right)p_{\alpha\perp}}{nm_{\alpha}}\mathbf{B}\cdot\nabla\ln B = 0, \tag{9}$$

$$\frac{d\mathbf{B}}{dt} + \mathbf{B} div \mathrm{v} - (\mathbf{B}\cdot\nabla)\,\mathrm{v} = 0, \quad div\mathbf{B} = 0. \tag{10}$$

In these equations $\frac{d}{dt} = \frac{\partial}{\partial t} + \mathrm{v}\cdot\nabla$, $\rho_{\alpha} = nm_{\alpha}$, $\alpha = \{e, i\}$ and $S_{\alpha\parallel}$ and $S_{\alpha\perp}$ are heat fluxes along magnetic field caused by longitudinal and transverse heat motions of particles of α type. If these fluxes are neglected, i.e. $S_{\parallel} = 0, S_{\perp} = 0$, we get the laws of time evaluten the longitudinal and transverse heat energy along a fluid element trajectory (the left hand sides of the equations (6) and (7)). The pair of equations (so-called double "adiabats") and equations (4),(5) and (10)

form the closed system of the equations known as C.G.L. equations [3]. However, if to use C.G.L. equations, the equations (8) and (9) will remain unsatisfied. It is a consequence of the fact that in deriving C.G.L. equations without any reasons the third moments of distribution function are omitted, i.e. heat fluxes are not considered. The equations (4)-(10) include heat fluxes and are more complete ones. C.G.L. equations don't follow from these equations as a special case.

3. DERIVATION OF DISPERSION EQUATION

For simplicity we will consider the case when unperturbed state of plasma is homogeneous and stationary: all values of $\mathrm{v}_0, \rho_0, p_{\|0}, p_{\perp 0}, \mathbf{B}_0, S_{\|0}$ and $S_{\perp 0} \to$ for α type particles don't depend on coordinates and time. The equations (4)-(10) automatically satisfy this background with nonzero heat fluxes. Let's consider the small perturbations of physical variables relative to equilibrium state. For example, we will present the pressure in the form of $p = p_0 + p'(r,t)$, where $p'(r,t) \sim \exp i(\mathbf{k}\cdot\mathbf{r} - \omega_0 t)$ and $|p'| \ll p_0$. Here $\omega_0 = \omega + (\mathrm{v}_0 \cdot \mathbf{k})$ is an oscillation frequency in the coordinate system moving with plasma, $\mathbf{k}-$ wave vector of oscillations. For perturbations in the linear approximation we get the following system of equations:

$$\rho'_\alpha \omega - \rho_{\alpha 0}\left(k_x \mathrm{v}'_x + k_y \mathrm{v}'_y + k_z \mathrm{v}'_z\right) = 0, \tag{11}$$

$$\begin{gathered}\omega\rho_0 \mathrm{v}' + \frac{1}{4\pi}(\mathbf{k}\cdot\mathbf{B_0})\mathbf{B}' - \frac{1}{4\pi}B_0 B'\mathbf{k} - \sum_\alpha \Bigg[\mathbf{k}p'_{\alpha\perp} + \frac{p_{\alpha\|0} - p_{\alpha\perp 0}}{B_0^2}\times \\ \times(\mathbf{k}\cdot\mathbf{B}_0)\left(\mathbf{B}' - 2\frac{B'}{B_0}\mathbf{B_0} + \frac{p'_{\alpha\|} - p'_{\alpha\perp}}{p_{\alpha\|0} - p_{\alpha\perp 0}}\mathbf{B_0}\right)\Bigg] = 0,\end{gathered} \tag{12}$$

$$2\frac{B'}{B_0} + \frac{p'_{\alpha\perp}}{p_{\alpha\perp 0}} - 3\frac{\rho'_\alpha}{\rho_{\alpha 0}} - \frac{S_{\alpha\|0}}{p_{\alpha\|0}\omega}\left(\frac{\mathbf{B}_0}{B_0}\cdot\mathbf{k}\right)\left(\frac{2S_{\alpha\perp 0} - S_{\alpha\|0}}{S_{\alpha\|0}}\frac{B'}{B_0} + \frac{S'_{\alpha\|}}{S_{\alpha\|0}}\right) = 0, \tag{13}$$

$$\frac{p'_{\alpha\perp}}{p_{\alpha\perp 0}} - \frac{\rho'_\alpha}{\rho_{\alpha 0}} - \frac{B'}{B_0} - \frac{S_{\alpha\perp 0}}{p_{\alpha\perp 0}\omega}\left(\frac{\mathbf{B}_0}{B_0}\cdot\mathbf{k}\right)\left(\frac{S'_{\alpha\perp}}{S_{\alpha\perp 0}} - 2\frac{B'}{B_0}\right) = 0, \tag{14}$$

$$\frac{S'_{\alpha\|}}{S_{\alpha\|0}} + 3\frac{B'}{B_0} - 4\frac{\rho'_\alpha}{\rho_{\alpha 0}} - \frac{3p_{\alpha\|0}^2}{S_{\alpha\|0}\rho_{\alpha 0}\omega}\left(\frac{\mathbf{B}_0}{B_0}\cdot\mathbf{k}\right)\left(\frac{p'_{\alpha\|}}{p_{\alpha\|0}} - \frac{\rho'_\alpha}{\rho_{\alpha 0}}\right) = 0, \tag{15}$$

$$\frac{S'_{\alpha\perp}}{S_{\alpha\perp 0}} - 2\frac{\rho'_\alpha}{\rho_{\alpha 0}} - \frac{p_{\alpha\|0}p_{\alpha\perp 0}}{S_{\alpha\perp 0}\rho_{\alpha 0}\omega}\left(\frac{\mathbf{B}_0}{B_0}\cdot\mathbf{k}\right)\left(\frac{p'_{\alpha\perp}}{p_{\alpha\perp 0}} - \frac{\rho'_\alpha}{\rho_{\alpha 0}} - \frac{p_{\alpha\|0} - p_{\alpha\perp 0}}{p_{\alpha\|0}}\frac{B'}{B_0}\right) = 0, \tag{16}$$

$$\omega\mathbf{B}' - \mathbf{B}_0(\mathbf{k}\cdot\mathrm{v}) + (\mathbf{B}_0\cdot\mathbf{k})\,\mathrm{v} = 0, \tag{17}$$

$$\left(\mathbf{k}\cdot\mathbf{B}'\right) = 0. \tag{18}$$

Eliminating the perturbations of heat fluxes from the equations

$$S'_{\alpha\perp} = \frac{k_\| p_{\alpha\|0} p_{\alpha\perp 0}}{\omega \rho_{\alpha 0}} \left(\frac{p'_{\alpha\perp}}{p_{\alpha\perp 0}} - \frac{\rho'_\alpha}{\rho_{\alpha 0}} - \frac{p_{\alpha\|0} - p_{\alpha\perp 0}}{p_{\alpha\|0}} \frac{B'}{B_0} \right) + 2 S_{\alpha\perp 0} \frac{\rho'_\alpha}{\rho_{\alpha 0}}, \tag{19}$$

$$S'_{\alpha\|} = \frac{3 p_{\alpha\|0}^2 k_\|}{\omega \rho_{\alpha 0}} \left(\frac{p'_{\alpha\|}}{p_{\alpha\|0}} - \frac{\rho'_\alpha}{\rho_{\alpha 0}} \right) - S_{\alpha\|0} \left(3 \frac{B'}{B_0} - 4 \frac{\rho'_\alpha}{\rho_{\alpha 0}} \right), \tag{20}$$

we get the system of equations

$$\rho'_\alpha \omega - \rho_{\alpha 0} \left(k_x \mathrm{v}'_x + k_y \mathrm{v}'_y + k_z \mathrm{v}'_z \right) = 0, \tag{21}$$

$$a_{\alpha 0} \frac{p'_{\alpha\perp}}{p_{\alpha\perp 0}} = a_{\alpha 1} \frac{B'}{B_0} + a_{\alpha 2} \frac{\rho'_\alpha}{\rho_{\alpha 0}}; b_{\alpha 0} \frac{p'_{\alpha\|}}{p_{\alpha\|0}} = b_{\alpha 1} \frac{B'}{B_0} + b_{\alpha 2} \frac{\rho'_\alpha}{\rho_{\alpha 0}}, \tag{22}$$

$$\begin{gathered} \omega \rho_0 \mathrm{v}' + \frac{1}{4\pi} \left(\mathbf{k} \cdot \mathbf{B_0} \right) \mathbf{B}' - \frac{1}{4\pi} B_0 B' \mathbf{k} - \sum_\alpha \Bigg[\mathbf{k} p'_{\alpha\perp} + \frac{p_{\alpha\|0} - p_{\alpha\perp 0}}{B_0^2} \times \\ \times \left(\mathbf{k} \cdot \mathbf{B}_0 \right) \left(\mathbf{B}' - 2 \frac{B'}{B_0} \mathbf{B_0} + \frac{p'_{\alpha\|} - p'_{\alpha\perp}}{p_{\alpha\|0} - p_{\alpha\perp 0}} \mathbf{B_0} \right) \Bigg] = 0, \end{gathered} \tag{23}$$

$$\omega \mathbf{B}' - \mathbf{B}_0 \left(\mathbf{k} \cdot \mathrm{v} \right) + \left(\mathbf{B}_0 \cdot \mathbf{k} \right) \mathrm{v} = 0, \quad \left(\mathbf{k} \cdot \mathbf{B}' \right) = 0. \tag{24}$$

Here

$$\begin{gathered} a_{\alpha 0} = 1 - \eta_\alpha^2, a_{\alpha 1} = 1 - 2\gamma_\alpha \eta_\alpha - \bar{\varphi}_\alpha \eta_\alpha^2, a_{\alpha 2} = 1 + 2\gamma_\alpha \eta_\alpha - \eta_\alpha^2, \\ b_{\alpha 0} = 1 - 3\eta_\alpha^2, b_{\alpha 1} = 2\gamma_\alpha \eta_\alpha \left(\varphi_\alpha - 2 \right) - 2, b_{\alpha 2} = 3 + 4\gamma_\alpha \eta_\alpha - 3\eta_\alpha^2. \end{gathered} \tag{25}$$

For the main unperturbed state we introduce the definitions (zero indexes are omitted):

$$\begin{gathered} \rho \approx \rho_i = n m_i, \ \varphi_\alpha = \frac{p_{\alpha\perp}}{p_{\alpha\|}}, \ \bar{\varphi}_\alpha = 1 - \varphi_\alpha, \ c_{\alpha\|}^2 = \frac{p_{\alpha\|}}{\rho_\alpha}, \ \beta_\alpha = \frac{B^2}{4\pi p_{\alpha\|}}, \\ \eta_\alpha \equiv \frac{c_{\alpha\|} k_\|}{\omega} = \frac{c_{\alpha\|} k}{\omega} \cos\theta, \ \bar{S}_{\alpha\|} = \frac{S_{\alpha\|}}{p_{\alpha\|} c_{\alpha\|}}, \ \bar{S}_{\alpha\perp} = \frac{S_{\alpha\perp}}{p_{\alpha\perp} c_{\alpha\|}}, \\ l = l_1 = \cos^2\theta, \ l_2 = \sin^2\theta, \ k_\| = \frac{\mathbf{B}_0}{B_0} \cdot \mathbf{k}, \ c_A^2 \equiv \frac{B^2}{4\pi\rho}, \\ \gamma_\alpha = \bar{S}_{\alpha\|} = \bar{S}_{\alpha\perp}, S_{\alpha\perp 0} \approx \frac{3}{4} u_0 \sigma p_{\alpha\perp}, \ S_{\alpha\|0} \approx \frac{3}{4} u_0 \sigma p_{\alpha\|}, \\ p_{\alpha\|} = \frac{1}{2} n_e k_B T_{\alpha\|}, c_{\alpha\perp}^2 = \frac{p_{\alpha\perp}}{\rho_\alpha}, \ \Omega \equiv \frac{c_{i\|}}{c_{e\|}} = \frac{\psi}{\sqrt{\Lambda_e}}, \\ \Lambda_\alpha = \frac{m_i + m_e}{m_\alpha} \approx \frac{m_i}{m_\alpha}, \ p_{i\|} = \Lambda_e \Omega^2 p_{e\|}, \ \gamma_\alpha \approx \frac{u_0}{c_{\alpha\|}}, \ \Lambda_e = \kappa^2 \approx 1836, \\ \psi^2 = \frac{p_{i\|}}{p_{e\|}} = \frac{T_{i\|}}{T_{e\|}} = \frac{1}{\tau_\|}, c_{s\perp}^2 \equiv \left(\sum_\alpha p_{\alpha\perp} \right) / \rho, \ c_{s\|}^2 \equiv \left(\sum_\alpha p_{\alpha\|} \right) / \rho. \end{gathered} \tag{26}$$

Supposing without loss of generality of formulation of the problem

$$\mathbf{k} = (k, 0, 0)\,, \quad \mathbf{B_0} = (B_{0x}, 0, B_{0z})\,, \\ B_{0x} = B_0 \cos\theta, \quad B_{0z} = B_0 \sin\theta, \tag{27}$$

we get

$$\frac{\mathbf{B}'}{B} - \frac{\mathbf{B}}{B}\frac{(\mathbf{k}\cdot \mathrm{v})}{\omega} + \frac{k_\parallel}{\omega}\mathrm{v} = 0, \tag{28}$$

$$\begin{gathered}\frac{1}{\eta_i}\frac{\mathrm{v}}{c_{i\parallel}} - \varphi_i \frac{\mathbf{k}}{k_\parallel}\left(\frac{a_{i1}}{a_{i0}}\frac{B'}{B} + \frac{a_{i2}}{a_{i0}}\frac{(\mathbf{k}\cdot\mathrm{v})}{\omega} + \frac{(\mathbf{B}\cdot\mathbf{B}')}{4\pi p_{i\perp}}\right) + \frac{\mathbf{B}'}{B}\left(\beta_i - \bar{\varphi}_i\right) - \\ -\frac{\mathbf{B}}{B}\left[\left(\frac{b_{i1}}{b_{i0}} - \varphi_i\frac{a_{i1}}{a_{i0}} - 2\bar{\varphi}_i\right)\frac{B'}{B} + \left(\frac{b_{i2}}{b_{i0}} - \varphi_i\frac{a_{i2}}{a_{i0}}\right)\frac{(\mathbf{k}\cdot\mathrm{v})}{\omega}\right] - \\ -\tau_\parallel\left[\varphi_e\frac{\mathbf{k}}{k_\parallel}\left(\frac{a_{e1}}{a_{e0}}\frac{B'}{B} + \frac{a_{e2}}{a_{e0}}\frac{(\mathbf{k}\cdot\mathrm{v})}{\omega}\right) + \frac{\mathbf{B}'}{B}\bar{\varphi}_e + \right. \\ \left. +\frac{\mathbf{B}}{B}\left[\left(\frac{b_{e1}}{b_{e0}} - \varphi_e\frac{a_{e1}}{a_{e0}} - 2\bar{\varphi}_e\right)\frac{B'}{B} + \left(\frac{b_{e2}}{b_{e0}} - \varphi_e\frac{a_{e2}}{a_{e0}}\right)\frac{(\mathbf{k}\cdot\mathrm{v})}{\omega}\right]\right] = 0.\end{gathered} \tag{29}$$

Here

$$\begin{gathered} q_{i4} = \beta_i - \bar{\varphi}_i - l_2 q_{i0}, \quad q_{e4} = -\bar{\varphi}_e - l_2 q_{e0}, \\ q_{i2} = \varphi_i\frac{a_{i1}}{a_{i0}} + \beta_i + l_1 q_{i0}, \quad q_{e2} = \varphi_e\frac{a_{e1}}{a_{e0}} + l_1 q_{e0}, \\ q_{i1} = \frac{1}{\eta_i} - \eta_i\frac{\varphi_i}{l_1}\frac{a_{i2}}{a_{i0}} - q_{i3}, \quad q_{e1} = -\eta_e\frac{\varphi_e}{l_1}\frac{a_{e2}}{a_{e0}} - q_{e3}, \\ q_{i3} = \eta_i\left(\frac{b_{i2}}{b_{i0}} - \varphi_i\frac{a_{i2}}{a_{i0}}\right), \quad q_{e3} = \eta_e\left(\frac{b_{e2}}{b_{e0}} - \varphi_e\frac{a_{e2}}{a_{e0}}\right), \\ q_{i0} = \frac{b_{i1}}{b_{i0}} - \varphi_i\frac{a_{i1}}{a_{i0}} - 2\bar{\varphi}_i, \quad q_{e0} = \frac{b_{e1}}{b_{e0}} - \varphi_e\frac{a_{e1}}{a_{e0}} - 2\bar{\varphi}_e. \end{gathered} \tag{30}$$

$y-$ component of this system is reduced to

$$\begin{gathered} \frac{B'_y}{B} + \eta_i\frac{\mathrm{v}_y}{c_{i\parallel}} + \frac{B'_y}{B} + \eta_e\frac{\mathrm{v}_y}{c_{e\parallel}} = 0, \\ p_{i\parallel}\frac{1}{\eta_i}\frac{\mathrm{v}_y}{c_{i\parallel}} + p_{i\parallel}\left(\beta_i - \bar{\varphi}_i\right)\frac{B'_y}{B} - p_{e\parallel}\bar{\varphi}_e\frac{B'_y}{B} = 0, \end{gathered} \tag{31}$$

from which we may easily derive the dispersion equation for the incompressible modes:

$$\sum_\alpha \frac{\eta_\alpha^2}{\Lambda_\alpha}\left(\frac{1}{2}\beta_\alpha + \varphi_\alpha - 1\right) = 1. \tag{32}$$

In dimensional quantities this corresponds to

$$\omega^2 = c_A^2 k_\parallel^2\left(1 - 4\pi\sum_\alpha \frac{p_{\alpha\parallel} - p_{\alpha\perp}}{B^2}\right), \tag{33}$$

which is a dispersion equation of type of Alfvén oscillations in isotropic plasma. Other x − and z − components of the system of equations (28-29) take the form:

$$q_{i1}\frac{\mathrm{v}_x}{c_{i\|}} - q_{i2}\tan\theta\frac{B'_z}{B} + \tau_\| q_{e1}\frac{\mathrm{v}_x}{c_{e\|}} - \tau_\| q_{e2}\tan\theta\frac{B'_z}{B} = 0,$$
$$\frac{B'_z}{B} - \eta_\alpha\tan\theta\frac{\mathrm{v}_x}{c_{\alpha\|}} + \eta_\alpha\frac{\mathrm{v}_z}{c_{\alpha\|}} = 0, \tag{34}$$
$$\frac{1}{\eta_i}\frac{\mathrm{v}_z}{c_{i\|}} - q_{i3}\tan\theta\frac{\mathrm{v}_x}{c_{i\|}} + q_{i4}\frac{B'_z}{B} - \tau_\| q_{e3}\tan\theta\frac{\mathrm{v}_x}{c_{e\|}} + \tau_\| q_{e4}\frac{B'_z}{B} = 0.$$

Determinant of this system gives dispersion equation for compressible modes:

$$\sum_\alpha l_1\frac{\eta_\alpha q_{\alpha 1}}{\Lambda_\alpha}\left(1 - \sum_\alpha\frac{\eta_\alpha^2 q_{\alpha 4}}{\Lambda_\alpha}\right) - \sum_\alpha l_2\frac{\eta_\alpha^2 q_{\alpha 2}}{\Lambda_\alpha}\left(1 - \sum_\alpha\frac{\eta_\alpha q_{\alpha 3}}{\Lambda_\alpha}\right) = 0. \tag{35}$$

Since $\eta_i = \Omega\eta_e, c_{e\|}^2\rho_e = \dfrac{p_{i\|}}{\Lambda_e\Omega^2}, \eta_i = \dfrac{1}{x}$ dispersion equation for Alfvén modes has the form

$$\frac{\omega^2}{c_{i\|}^2 k^2} = l\left(\beta_i + \varphi_i - 1 + \frac{\varphi_e - 1}{\psi^2}\right). \tag{36}$$

When

$$\beta_i + \varphi_i + \varphi_e/\psi^2 < 1 + 1/\psi^2, \tag{37}$$

Alfvén modes become unstable, i.e. fire-hose instability occurs. In two cases increment of fire-hose instability turns to known case: when $\psi^2 \gg 1$ (cold electrons, $T_{e\|} \ll T_{i\|}$) and for the isotropic electrons $\varphi_e = 1$. If electron component of plasma is anisotropic i.e., $\varphi_e \neq 1$, and $\varphi_e > 1$ ($T_{e\perp} > T_{e\|}$) then fire-hose instability is suppressed.In opposite case, when $\varphi_e < 1$ ($T_{e\perp} < T_{e\|}$), conversely, instability intensifies.

Dispersion equation for compressible modes (35) is given as

$$\left(x^2 - 1\right)\left(x^2 - 3\right)\left(\Omega^2 x^2 - 3\right)\left(\Omega^2 x^2 - 1\right) Z = 0. \tag{38}$$

It may be easily proved that roots of multipliers $\left(x^2 - 1\right)\left(x^2 - 3\right)$ $\left(\Omega^2 x^2 - 3\right)\left(\Omega^2 x^2 - 1\right)$ are false (see in (22)). The last multiplier Z in (38) is dispersion equation for compressible modes. This is a polynomial equation of the 12th order for normalized phase of velocity $x = \omega/k_\| c_{i\|}$:

$$U_{12}x^{12} + U_{10}x^{10} + U_8x^8 + U_6x^6 + U_4x^4 + U_2x^2 + U_0 +$$
$$+\gamma_\alpha\left[U_9x^9 + U_7x^7 + U_5x^5 + U_3x^3 + U_1x\right] = 0, \tag{39}$$

coefficients $U_{0..12}$ are presented in the 5.

If to transfer to one-component ionic plasma $\psi \to \infty$, the general dispersion equation (39) turns to

$$C_8x^8 + C_6x^6 + C_4x^4 + C_2x^2 + C_0 + \gamma_i\left[C_5x^5 + C_3x^3 + C_1x\right] = 0, \tag{40}$$

$$\begin{gathered} s = \varphi_i^2\,(1-l)\,,\ r = l - \beta_i - \varphi_i\,(2-l)\,, \\ C_0 = 3\,(2s+r)\,,\ C_1 = -4\,(3s+r)\,,\ C_2 = 2s\left(2\gamma_i^2 - 5\right) + 3\,(l-3r)\,, \\ C_3 = 4\,(s+r-l)\,, C_4 = 2s + 7r - 9l, C_5 = 4l, C_6 = 7l - r, C_8 = -l, \end{gathered} \tag{41}$$

which exactly coincides with the results of the paper [6]. In general the equation (39) is complicated. It includes the effects of anisotropy of both ion and electron components. The interaction of ion and electron oscillations is complicated by existence of heat fluxes along magnetic field.

4. WAVES IN NON-FLUX CASE

For identification of wave modes we exclude the effects, related to heat fluxes. Let us to suppose $\gamma_\alpha = 0$. For simplicity we denote $t = x^2, \mathrm{v}_{ph}^2 \equiv \omega^2/c_{i\|}^2 k^2 = lt$. Then the equation (39) has the form

$$U_{12}t^6 + U_{10}t^5 + U_8t^4 + U_6t^3 + U_4t^2 + U_2t + U_0 = 0. \tag{42}$$

4.1. Parallel wave propagation

In case of parallel wave propagation relative to the magnetic field, $l = 1$, the equation (42) can be written in the form:

$$(t-1)\left(-\psi^2 t + \kappa^2\right)\left(\psi^2 t - \psi^2\beta_i - \psi^2\varphi_i + \psi^2 - \varphi_e + 1\right)\bar{Z} = 0, \tag{43}$$

where

$$\begin{gathered} \bar{Z} \equiv -\psi^4 t^3 + 3\,\kappa^2\psi^2 t^2 + 6\,\psi^4 t^2 - 18\,\kappa^2\psi^2 t - 3\,\psi^4 t + 9\,\kappa^2\psi^2 + \\ +3\,\psi^2 t^2 - 3\,\kappa^2 t - 9\,\psi^2 t + 9\,\kappa^2. \end{gathered} \tag{44}$$

In (43) the first solution $t = 1$ corresponds to slow ion- acoustic modes (SIA), for which $\omega^2 = c_{i\|}^2 k_\|^2$. The second multiplier $\left(-\psi^2 t + \kappa^2\right) = 0$ describes slow electron-acoustic modes (SEA), for which $\omega^2 = c_{e\|}^2 k_\|^2$. The third multiplier $\psi^2 t - \psi^2\beta_i - \psi^2\varphi_i + \psi^2 - \varphi_e + 1 = 0$ in (43) describes fast magnetosonic modes (FMS), for which $\omega^2 = c_{i\|}^2 k_\|^2\left(\varphi_i\psi^2 + \beta_i\psi^2 - \psi^2 + \varphi_e - 1\right)/\psi^2$, which can be represented as $\omega^2 = c_A^2 k_\|^2\left(1 - 4\pi\sum_\alpha \frac{p_{\alpha\|} - p_{\alpha\perp}}{B^2}\right)$. Fast magnetosonic modes in

parallel propagation case become uncompressible and dispersion equation coincides with Alfvén ones (33).

The fourth multiplier $\bar{Z} = 0$ in (43) is a cubic equation. We will not provide complex exact formulas for the solution of this cubic equation. We will use the expansion for the small parameter $1/\kappa^2 = \epsilon \ll 1$. When we suppose $\psi^2 = \mu$, then

$$\begin{gathered}\bar{Z} \equiv -\epsilon\mu t\left[\mu t^2 - 3\left(2\mu+1\right)t + 3\left(\mu+3\right)\right] + \\ +3\left[\mu t^2 - \left(6\mu+1\right)t + 3\left(\mu+1\right)\right] = 0.\end{gathered} \tag{45}$$

When $\epsilon \to 0$ we get

$$t_{1,2} = \frac{6\mu + 1 \pm \sqrt{\left(6\mu+1\right)^2 - 12\left(\mu+1\right)\mu}}{2\mu}, \tag{46}$$

or

$$\omega^2 = \left[\frac{6 + \tau_\parallel \pm \sqrt{24 + \tau_\parallel^2}}{2}\right] c_{i\parallel}^2 k_\parallel^2. \tag{47}$$

Here the first solution (upper sign) describes slow sonic modes (SS). This solution can be written as

$$\omega^2 = \left[\frac{6 + \tau_\parallel + \sqrt{24 + \tau_\parallel^2}}{2\left(1 + \tau_\parallel\right)}\right] c_{s\parallel}^2 k_\parallel^2. \tag{48}$$

At $\tau_\parallel \to 1$(isothermal plasma) $\omega^2 = 3c_{s\parallel}^2 k_\parallel^2$, but at $\tau_\parallel \to \infty$ (cold ions) $\omega^2 = c_{s\parallel}^2 k_\parallel^2$. The second solution (lower sign) describes fast ion-acoustic modes (FIA)

$$\omega^2 = \left[\frac{6 + \tau_\parallel - \sqrt{24 + \tau_\parallel^2}}{2}\right] c_{i\parallel}^2 k_\parallel^2. \tag{49}$$

At $\tau_\parallel \to 1$ (isothermal plasma) $\omega^2 = c_{i\parallel}^2 k_\parallel^2$, but at $\tau_\parallel \to \infty$ (cold ions) $\omega^2 = 3c_{i\parallel}^2 k_\parallel^2$.

The third solution of (45)

$$oft_3 \approx \frac{1}{\epsilon} \cdot \frac{3}{\mu} = \frac{3\kappa^2}{\psi^2}, \tag{50}$$

that coincides with fast electron-acoustic modes (FEA), $\omega^2 = 3c_{e\parallel}^2 k_\parallel^2$.

Thus, in non-flux mode for parallel propagation the general dispersion equation (39) turns to (43), which describes six modes: fast magnetosonic, slow sonic, fast/slow ion-acoustic and fast/slow electron-acoustic modes. Among them only fast magnetosonic waves can be unstable during the development of fire-hose instability.

4.2. Perpendicular wave propagation

In the case of quasi-perpendicular wave propagation ($l \to 0$) from the dispersion equation (42) for fast magnetosonic waves we get

$$\omega^2 = k^2 \left(c_A^2 + 2c_{s\perp}^2\right). \tag{51}$$

For the other modes of oscillations $\omega^2 \sim l \to 0$.

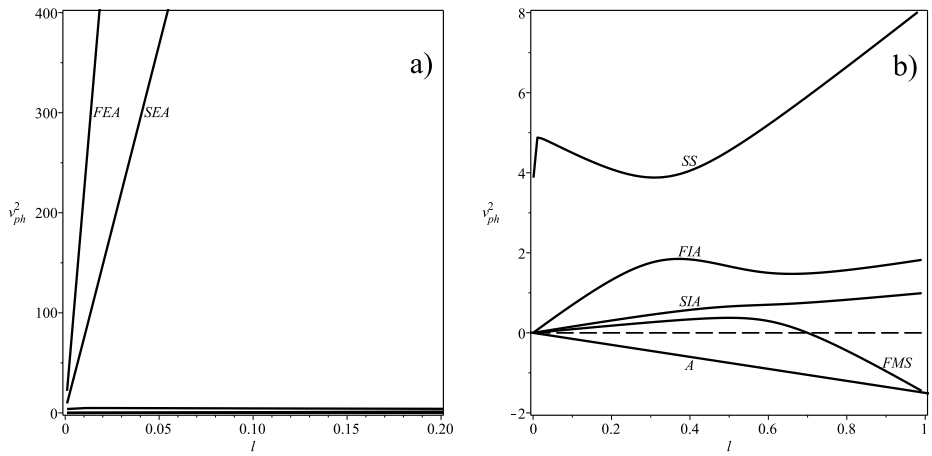

Fig. 1. Dependence of square of phase velocity of waves on parameter of propagation angle $l = cos^2\theta$ in case of fire-hose instability (37): $\varphi_e = \varphi_i = 0.5; \beta_i = 1; \psi = 0.5$. Instability occurs, if $\mathrm{v}_{ph}^2 < 0$. Alfvén (A) and fast magnetosonic (FMS) modes become unstable. The electron acoustic waves are shown on the left figure (a), all other wave branches are shown on the right figure (b).

4.3. Oblique propagation of waves

In general, when waves propagation taves place at arbitrary angles, $0 < l < 1$, dispersion equation (42) should be solved numerically. Coefficients of this equation are real functions of five parameters: $l-$ parameter of wave propagation angle relative to the direction of magnetic field; β_i- magnetic field intensity parameter; $\psi-$ rate of nonisothermality of plasma; φ_e and φ_i- anisotropy rate of electron and ion plasma components. Choosing a set of parameters we separate two characteristic cases, when fire$-$hose instability occurs (condition (37) is satisfied), and when fire$-$hose instability does not occur (condition (37) is not satisfied). Fig. 1 shows the dependence of normalized square of phase velocity v_{ph}^2 on l. On curves the names of wave modes were indicated which we established in previous section for longitudinal propagation. The instability occurs in those areas

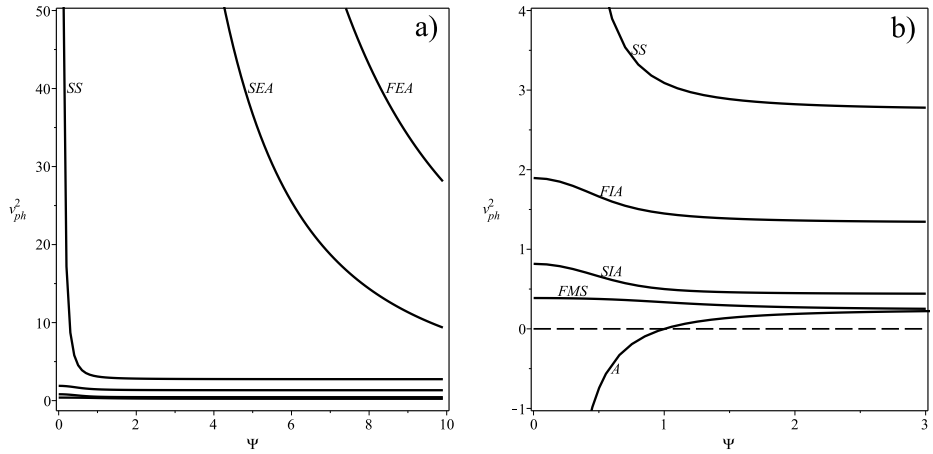

Fig. 2. . Dependence of square of phase velocity of waves on parameter of isothermality of plasma $\psi = (T_{i\|}/T_{e\|})^{1/2}$ in given $l = 0.5$. The condition of fire-hose instability (37) for $\varphi_e = \varphi_i = 0.5$ and $\beta_i = 1$ is fulfilled, if $\psi < 1$. The other modes are stable.

where $\mathrm{v}_{ph}^2 < 0$. For the fixed parameters in Fig. 1 Alfvén waves (A) for all angles become unstable (fire−hose instability), however the fast magnetosonic modes (FMS) become unstable only in case of quasi-longitudinal propagation. Threshold of instability of $FMS-$ modes is determined by $U_0(l, \psi, \beta_i, \varphi_e, \varphi_i) = 0$, where the function U_0 is defined in (66). For stable modes phase velocities are ordered as $A < FMS < SIA < FIA < SS < SEA < FEA$. In Fig. 2 the dependence of normalized square of phase velocity v_{ph}^2 on ψ is shown at the angle of propagation $l = 0.5$. As one can see, fire−hose instability for this set of parameters occurs only in Alfvén modes. According to this figure behavior of waves at small and large values of ψ considerably differs. We should note that the question of wave instability was studied earlier in details at $\psi \gg 1$(ion plasma). Under realistic situations, however, as a rule $\psi \sim O(1)$. In other set of parameters when condition of fire−hose instability (37) is not satisfied, mirror instability may occur at high angle propagation. Such example is shown in Fig. 3 where mirror instability develops on a branch of fast ion-acoustic modes (FIA).The threshold of mirror instability also exists. To define the criteria of appearance of mirror instability we will suppose $U_0 = \epsilon \to 0$. Then the equation (42) becomes

$$t(U_{12}t^5 + U_{10}t^4 + U_8t^3 + U_6t^2 + U_4t + U_2) + \epsilon = 0. \tag{52}$$

One of the solutions of this equation $t \approx \dfrac{\epsilon}{-U_2} \Rightarrow x^2 \approx -\dfrac{U_0}{U_2}$ defines the criterion of appearance of instability in general form: $U_0/U_2 > 0$. In simplest

case, when protons and electrons are equally anisotropic, $\varphi_i = \varphi_e = \varphi$, in quasi-perpendicular propagation passing to limit $l \to 0$ we get

$$x^2 \approx -\frac{3\left(\psi^2+1\right)\left[\psi^2\left(2\varphi^2-2\varphi-\beta_i\right)+2\varphi^2-2\varphi\right]}{\dfrac{16\varphi^3\beta_i\left(\varphi-1\right)}{\left(2\varphi^2-2\varphi-\beta_i\right)^2}}. \tag{53}$$

From this the condition of appearance of mirror instability is obtained

$$\frac{\psi^2\left(2\varphi^2-2\varphi-\beta_i\right)+2\varphi\left(\varphi-1\right)}{\varphi-1} > 0. \tag{54}$$

When $\varphi < 1$, this condition is always satisfied. If $\varphi > 1$ then $\psi^2\left[2\varphi\left(\varphi-1\right)-\beta_i\right]+2\varphi\left(\varphi-1\right) > 0$ should be fulfilled. When $\psi \to \infty$ (ion plasma) this condition goes into $2\varphi^2 > 2\varphi+\beta_i$ or $\dfrac{p_\perp^2}{p_\parallel} > p_\perp + p_m$, which is a classical condition of appearance of mirror instability [6]. For $\psi \to 1$ ($T_{e\parallel} = T_{i\parallel}$ isothermal plasma) we have $4\varphi^2 > 2\varphi+\beta_i$. For hot electrons ($\psi \to 0$) the condition (54) is satisfied in all cases.

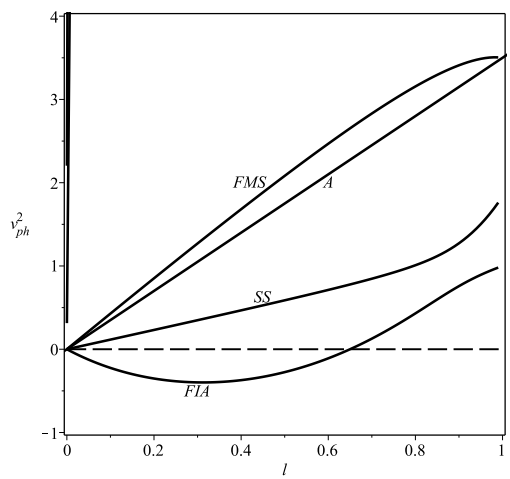

Fig. 3. Dependence of square of phase velocity of waves on the parameter of propagation angle $l = cos^2\theta$ for the lack of fire-hose instability (37):$\varphi_e = \varphi_i = 1.5; \beta_i = 1; \psi = 0.5$. In this case the mirror instability can develop in fast ion-acoustic (FIA) branch at quasi- perpendicular propagation.

5. DISCUSSIONS AND CONCLUSIONS

To study the reason of existence of large-scale wave turbulence observed in cosmic plasmas (solar and stellar coronae, solar and stellar winds, magnetospheres of an Earth and planets, heliosphere et al.) we considered wave instability an anisotropic homogeneous plasma. Obtained general dispersion polynomial equation of 14th order is the generalization of earlier studied one-fluid case to two-fluid one. Inclusion of electron and ion components of plasma considerably complicates the final result due to the interaction of wave branches of both components. Obtained equation includes the effects related to anisotropy of both components as well as effects of anisotropic heat fluxes. For simplification of analyses in this paper we analyzed the dispersion equation without considering heat fluxes in background. For longitudinal propagation of waves relative to magnetic field wave modes are separated and their dispersion equation exactly coincides with wave branches of electron and ion components known from kinetic consideration. It is shown that without heat flux in plasma only classical fire-hose and mirror instabilities may occur. Fire-hose instability occur in Alfvén and quasi-longitudinal fast magnetosonic modes. The presence of anisotropic electrons has strong influence on the appearance of fire-hose instability. If $T_{e\perp} < T_{e\|}$ then the instability is enhanced. In opposite case $T_{e\perp} > T_{e\|}$ this instability is suppressed. This instability entirely disappears, if $T_{e\perp} > T_{e\|} + T_{i\|}$. In realistic situations, for example, in heliosphere, the condition is opposite and fire-hose instability may occur. The second classical instability is compressible mirror instability. It is developed in case of at quasi-perpendicular propagation of ion-acoustic modes. The influence of electron component on criteria of appearance of mirror instability is essential. Depending on the value of ratio $T_{i\|}/T_{e\|}$ threshold of instability is changed. Modification of classical instabilities of anisotropic plasma with a heat flux, as well as other new types of heat instabilities will be studied in further works.

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APPENDIX 1

Coefficients in the main dispersion equation of (39) are

$$U_{12} = \psi^8 l, \tag{55}$$

$$U_{10} = \psi^8 l\varphi_i - 4\,\psi^6 l\kappa^2 - 6\,\psi^8 l - \psi^8\beta_i - 2\,\psi^8\varphi_i + \psi^6 l\varphi_e - 2\,\psi^6 l - 2\,\psi^6\varphi_e, \tag{56}$$

$$\gamma_\alpha U_9 = -4\,\psi^8\gamma_i l - 4\,\psi^5\gamma_e l\kappa, \tag{57}$$

$$\begin{gathered} U_8 = 2\,\psi^8 l{\varphi_i}^2 - 4\,\psi^6 l\varphi_i\kappa^2 - 7\,\psi^8 l\varphi_i - 2\,\psi^8{\varphi_i}^2 + 3\,\psi^4 l\kappa^4 + 24\,\psi^6 l\kappa^2 + \\ +\psi^4 l{\varphi_e}^2\kappa^2 + 4\,\psi^6\beta_i\kappa^2 + 8\,\psi^6\varphi_i\kappa^2 + 2\,\psi^8 l + 2\,\psi^6 l\varphi_e\varphi_i + 7\,\psi^8\beta_i + \\ +14\,\psi^8\varphi_i - 4\,\psi^4 l\varphi_e\kappa^2 - \psi^4{\varphi_e}^2\kappa^2 - 7\,\psi^6 l\varphi_e - 3\,\psi^6 l\varphi_i - 2\,\psi^6\varphi_e\varphi_i + \\ +2\,\psi^4 l\kappa^2 + 8\,\psi^4\varphi_e\kappa^2 + 2\,\psi^6 l + \psi^4 l{\varphi_e}^2 + 3\,\psi^6\beta_i + 14\,\psi^6\varphi_e + 6\,\psi^6\varphi_i - \\ -3\,\psi^4 l\varphi_e - \psi^4{\varphi_e}^2 - 3\,\psi^4 l + 6\,\psi^4\varphi_e, \end{gathered} \tag{58}$$

$$\begin{gathered} \gamma_\alpha\, U_7 = -4\,\psi^5\gamma_e l\varphi_i\kappa - 4\,\psi^5\gamma_e\varphi_e\varphi_i\kappa + 4\,\psi^3\gamma_e l{\varphi_e}^2\kappa - 4\,\psi^3\gamma_e l\varphi_e\kappa + \\ +4\,\psi^6\gamma_i l\varphi_e\varphi_i - 4\,\psi^6\gamma_i l\varphi_e - 4\,\psi^6\gamma_i\varphi_e\varphi_i - 4\,\psi^8\gamma_i l\varphi_i + 4\,\psi^8\gamma_i l{\varphi_i}^2 + \\ +4\,\psi^8\beta_i\gamma_i - 4\,\psi^6\gamma_i l + 8\,\psi^6\gamma_i\varphi_e + 8\,\psi^8\gamma_i\varphi_i - 4\,\psi^8\gamma_i{\varphi_i}^2 - 4\,\psi^3\gamma_e{\varphi_e}^2\kappa - \\ -4\,\psi^3\gamma_e l\kappa + 8\,\psi^3\gamma_e\varphi_e\kappa + 12\,\psi^5\gamma_e l\kappa + 8\,\psi^5\gamma_e\varphi_i\kappa + 16\,\psi^6\gamma_i l\kappa^2 + \\ +4\,\psi^5\beta_i\gamma_e\kappa + 4\,\psi^3\gamma_e l\kappa^3 + 4\,\psi^5\gamma_e l\varphi_e\varphi_i\kappa, \end{gathered} \tag{59}$$

$$\begin{gathered} U_6 = -8\,\psi^5\gamma_e\gamma_i\varphi_e\varphi_i\kappa + 7\,\psi^2{\varphi_e}^2\kappa^2 + 6\,\psi^2 l\kappa^2 - 12\,\psi^2\varphi_e\kappa^2 - 3\,\psi^6 l{\varphi_i}^2 - \\ 4\,\psi^8{\gamma_i}^2{\varphi_i}^2 - 6\,\psi^4\beta_i\kappa^2 + 8\,\psi^6{\varphi_i}^2\kappa^2 - 12\,\psi^4\varphi_i\kappa^2 + 3\,\psi^4 l\varphi_i\kappa^4 - \\ -3\,\psi^2 l{\varphi_e}^2\kappa^4 + 3\,\psi^2 l\varphi_e\kappa^4 + 9\,\psi^6 l\varphi_e + 9\,\psi^8 l\varphi_i - 8\,\psi^6 l\kappa^2 + \\ +8\,\psi^6\varphi_e\varphi_i + 12\,\psi^6 l\varphi_i + 12\,\psi^4 l\varphi_e - 4\,\psi^4 l{\varphi_e}^2 - 10\,\psi^8 l{\varphi_i}^2 + \\ +7\,\psi^4{\varphi_e}^2\kappa^2 - 28\,\psi^6\beta_i\kappa^2 - 56\,\psi^6\varphi_i\kappa^2 - 56\,\psi^4\varphi_e\kappa^2 + 10\,\psi^4 l\kappa^2 + \\ +12\,\psi^4 l + 3\,\psi^6{\varphi_i}^2 - 24\,\psi^4\varphi_e + 10\,\psi^8{\varphi_i}^2 - 12\,\psi^6\beta_i + 4\,\psi^4{\varphi_e}^2 - 24\,\psi^6\varphi_i + \\ +12\,\psi^6 l - 18\,\psi^8\varphi_i - 9\,\psi^8\beta_i + 6\,\psi^8 l + 4\,\psi^8{\gamma_i}^2 l{\varphi_i}^2 - \\ -4\,\psi^2{\gamma_e}^2{\varphi_e}^2\kappa^2 + 6\,\psi^2 l\varphi_e\kappa^2 + 6\,\psi^4 l\varphi_i\kappa^2 + 8\,\psi^4\varphi_e\varphi_i\kappa^2 - \\ -7\,\psi^2 l{\varphi_e}^2\kappa^2 - 8\,\psi^6 l{\varphi_i}^2\kappa^2 - 8\,\psi^6 l\varphi_e\varphi_i + 28\,\psi^4 l\varphi_e\kappa^2 - 7\,\psi^4 l{\varphi_e}^2\kappa^2 + \\ +28\,\psi^6 l\varphi_i\kappa^2 - 8\,\psi^4 l\varphi_e\varphi_i\kappa^2 + 4\,\psi^2{\gamma_e}^2 l{\varphi_e}^2\kappa^2 - 3\,\psi^4\beta_i\kappa^4 + 3\,\psi^2{\varphi_e}^2\kappa^4 - \\ -18\,\psi^4 l\kappa^4 - 6\,\psi^4\varphi_i\kappa^4 - 18\,\psi^6\varphi_e - 6\,\psi^2\varphi_e\kappa^4 + 8\,\psi^5\gamma_e\gamma_i l\varphi_e\varphi_i\kappa, \end{gathered} \tag{60}$$

$$
\begin{gathered}
\gamma_\alpha U_5 = -16\,\psi^5\gamma_e l\varphi_e\varphi_i\kappa - 8\,\psi^3\gamma_e l\varphi_e\varphi_i\kappa^3 - 16\,\psi^4\gamma_i l\varphi_e\varphi_i\kappa^2 + 4\,\psi^8\gamma_i l - \\
-4\,\psi^8\beta_i\gamma_i + 4\,\psi^6\gamma_i l - 8\,\psi^8\gamma_i\varphi_i + 12\,\psi^8\gamma_i{\varphi_i}^2 + 4\,\psi\,\gamma_e l\kappa^3 - \\
-8\,\psi\,\gamma_e\varphi_e\kappa^3 - 32\,\psi^4\gamma_i\varphi_e\kappa^2 + 16\,\psi^4\gamma_i l\kappa^2 + 16\,\psi^6\gamma_i{\varphi_i}^2\kappa^2 - \\
-32\,\psi^6\gamma_i\varphi_i\kappa^2 + 4\,\psi^4\gamma_i{\varphi_e}^2\kappa^2 + 4\,\psi^5\gamma_e{\varphi_i}^2\kappa - \\
-16\,\psi^6\beta_i\gamma_i\kappa^2 - 8\,\psi^3\gamma_e\varphi_i\kappa^3 - 12\,\psi^4\gamma_i l\kappa^4 + 12\,\psi\,\gamma_e{\varphi_e}^2\kappa^3 + 4\,\psi^6\gamma_i l\varphi_e + \\
+8\,\psi^6\gamma_i\varphi_e\varphi_i + 4\,\psi^8\gamma_i l\varphi_i - 12\,\psi^8\gamma_i l{\varphi_i}^2 + 16\,\psi^3\gamma_e{\varphi_e}^2\kappa + 16\,\psi^3\gamma_e l\kappa - \\
-32\,\psi^3\gamma_e\varphi_e\kappa - 32\,\psi^5\gamma_e\varphi_i\kappa - 16\,\psi^5\beta_i\gamma_e\kappa - 12\,\psi^3\gamma_e l\kappa^3 + 16\,\psi^6\gamma_i l\varphi_i\kappa^2 - \\
-16\,\psi^6\gamma_i l{\varphi_i}^2\kappa^2 - 4\,\psi^4\gamma_i l{\varphi_e}^2\kappa^2 + 4\,\psi^3\gamma_e l\varphi_i\kappa^3 + 8\,\psi^3\gamma_e\varphi_e\varphi_i\kappa^3 + \\
+16\,\psi^4\gamma_i l\varphi_e\kappa^2 + 16\,\psi^4\gamma_i\varphi_e\varphi_i\kappa^2 + 4\,\psi\,\gamma_e l\varphi_e\kappa^3 - \\
-12\,\psi\,\gamma_e l{\varphi_e}^2\kappa^3 + 16\,\psi^5\gamma_e l\varphi_i\kappa + 16\,\psi^5\gamma_e\varphi_e\varphi_i\kappa - 16\,\psi^3\gamma_e l{\varphi_e}^2\kappa + \\
+16\,\psi^3\gamma_e l\varphi_e\kappa - 8\,\psi^6\gamma_i l\varphi_e\varphi_i - 4\,\psi^5\gamma_e l{\varphi_i}^2\kappa - 8\,\psi^6\gamma_i\varphi_e + \\
+4\,\psi^5\gamma_e l\kappa - 4\,\psi^3\beta_i\gamma_e\kappa^3,
\end{gathered}
\tag{61}
$$

$$
\begin{gathered}
U_4 = -6\,\psi^4\varphi_i^2\kappa^2 + 16\,\psi^5\gamma_e\gamma_i\varphi_e\varphi_i\kappa + 16\,\psi^3\gamma_e\gamma_i\varphi_e\varphi_i\kappa^3 - 28\,\psi^2\varphi_e^2\kappa^2 - \\
-24\,\psi^2 l\kappa^2 + 9\,\psi^6 l\varphi_i^2 + 24\,\psi^4\beta_i\kappa^2 - 40\,\psi^6\varphi_i^2\kappa^2 + 48\,\psi^4\varphi_i\kappa^2 - \\
-21\,\psi^4 l\varphi_i\kappa^4 + 21\,\psi^2 l\varphi_e^2\kappa^4 - 21\,\psi^2 l\varphi_e\kappa^4 - 3\,\psi^2 l\varphi_i\kappa^4 - 3\,\psi^6 l\varphi_e - \\
-3\,\psi^8 l\varphi_i - 24\psi^6 l\kappa^2 - 6\,\psi^6\varphi_e\varphi_i - 9\,\psi^6 l\varphi_i - 9\,\psi^4 l\varphi_e + 3\,\psi^4 l{\varphi_e}^2 + \\
+6\,\psi^8 l{\varphi_i}^2 - 9\,\psi^4{\varphi_e}^2\kappa^2 + 36\,\psi^6\beta_i\kappa^2 + 72\,\psi^6\varphi_i\kappa^2 + 72\,\psi^4\varphi_e\kappa^2 - \\
-42\,\psi^4 l\kappa^2 + 6\,\psi^4 l{\varphi_i}^2\kappa^4 - 6\,\psi^2\varphi_e\varphi_i\kappa^4 - 9\,\psi^4 l + 6\,\psi^2 l\varphi_e\varphi_i\kappa^4 - \\
-9\,\psi^6{\varphi_i}^2 + 18\,\psi^4\varphi_e - 6\,\psi^8{\varphi_i}^2 + 9\,\psi^6\beta_i - 3\,\psi^4{\varphi_e}^2 + 18\,\psi^6\varphi_i - \\
-12\,\psi^6 l + 6\,\psi^8\varphi_i + 3\,\psi^8\beta_i + 6\,\psi^6\varphi_e - 3\,\psi^8 l + 6\,\psi^4 l{\varphi_i}^2\kappa^2 + \\
+16\,\psi^6{\gamma_i}^2{\varphi_i}^2\kappa^2 + 16\,\psi^2{\gamma_e}^2{\varphi_e}^2\kappa^2 - 24\,\psi^2 l\varphi_e\kappa^2 - 24\,\psi^4 l\varphi_i\kappa^2 + \\
+28\,\psi^2 l{\varphi_e}^2\kappa^2 + 40\,\psi^6 l{\varphi_i}^2\kappa^2 + 6\,\psi^6 l\varphi_e\varphi_i - 36\,\psi^4 l\varphi_e\kappa^2 + \\
+9\,\psi^4 l{\varphi_e}^2\kappa^2 - 36\,\psi^6 l\varphi_i\kappa^2 - 16\,\psi^6{\gamma_i}^2 l{\varphi_i}^2\kappa^2 + 32\,\psi^4 l\varphi_e\varphi_i\kappa^2 - \\
-16\,\psi^2{\gamma_e}^2 l{\varphi_e}^2\kappa^2 + 6\,l{\varphi_e}^2\kappa^4 - 3\,l\varphi_e\kappa^4 + 21\,\psi^4\beta_i\kappa^4 - \\
-21\,\psi^2{\varphi_e}^2\kappa^4 + 6\,\psi^4 l\kappa^4 + 42\,\psi^4\varphi_i\kappa^4 + 42\,\psi^2\varphi_e\kappa^4 - 12\,l\psi^2\kappa^4 + \\
+6\,\varphi_i\psi^2\kappa^4 + 3\,\beta_i\psi^2\kappa^4 - 6\,\psi^4{\varphi_i}^2\kappa^4 - 6\,{\varphi_e}^2\kappa^4 - 3\,l\kappa^4 + 6\,\varphi_e\kappa^4 - \\
-16\,\psi^5\gamma_e\gamma_i l\varphi_e\varphi_i\kappa - 16\,\psi^3\gamma_e\gamma_i l\varphi_e\varphi_i\kappa^3 + \\
+48\,\psi^2\varphi_e\kappa^2 - 32\,\psi^4\varphi_e\varphi_i\kappa^2,
\end{gathered}
\tag{62}
$$

$$
\begin{gathered}
\gamma_\alpha U_3 = 12\,\psi^4\gamma_i l{\varphi_i}^2\kappa^4 + 48\,\psi^6\gamma_i l{\varphi_i}^2\kappa^2 - 12\,\psi^4\gamma_i l\varphi_i\kappa^4 - 12\,\psi^4\gamma_i{\varphi_i}^2\kappa^4 - \\
-48\,\psi^6\gamma_i{\varphi_i}^2\kappa^2 + 12\,\psi^2\gamma_i l{\varphi_e}^2\kappa^4 + 12\,\psi^2\gamma_i l\varphi_e\varphi_i\kappa^4 + 12\,\psi^4\beta_i\gamma_i\kappa^4 + \\
+24\,\psi^4\gamma_i\varphi_i\kappa^4 + 32\,\psi^3\gamma_e l\varphi_e\varphi_i\kappa^3 + 4\,\psi^3\gamma_e l{\varphi_i}^2\kappa^3 - 16\,\psi^6\gamma_i l\kappa^2 + \\
+4\,\psi^4\gamma_i l{\varphi_e}^2\kappa^2 + 32\,\psi^4\gamma_i l\varphi_e\varphi_i\kappa^2 + 16\,\psi^6\beta_i\gamma_i\kappa^2 + \\
+32\,\psi^6\gamma_i\varphi_i\kappa^2 + 12\,\psi^5\gamma_e l\varphi_e\varphi_i\kappa + 12\,\psi^5\gamma_e l{\varphi_i}^2\kappa - 12\,\psi^2\gamma_i l\varphi_e\kappa^4 - \\
-12\,\psi^2\gamma_i{\varphi_e}^2\kappa^4 - 12\,\psi^2\gamma_i\varphi_e\varphi_i\kappa^4 - 16\,\psi^3\gamma_e l\varphi_i\kappa^3 - 32\,\psi^3\gamma_e\varphi_e\varphi_i\kappa^3 - \\
-4\,\psi^3\gamma_e{\varphi_i}^2\kappa^3 - 16\,\psi^4\gamma_i l\varphi_e\kappa^2 - 4\,\psi^4\gamma_i{\varphi_e}^2\kappa^2 - 32\,\psi^4\gamma_i\varphi_e\varphi_i\kappa^2 - \\
-12\,\psi^5\gamma_e l\varphi_i\kappa - 12\,\psi^5\gamma_e\varphi_e\varphi_i\kappa - 12\,\psi^5\gamma_e{\varphi_i}^2\kappa - 12\,\psi^2\gamma_i l\kappa^4 + \\
+24\,\psi^2\gamma_i\varphi_e\kappa^4 - 4\,\psi^3\gamma_e l\kappa^3 + 48\,\psi\,\gamma_e l{\varphi_e}^2\kappa^3 + \\
+16\,\psi^3\beta_i\gamma_e\kappa^3 + 32\,\psi^3\gamma_e\varphi_i\kappa^3 - 16\,\psi^4\gamma_i l\kappa^2 + 32\,\psi^4\gamma_i\varphi_e\kappa^2 - 12\,\psi^5\gamma_e l\kappa + \\
+12\,\psi^3\gamma_e l{\varphi_e}^2\kappa + 12\,\psi^5\beta_i\gamma_e\kappa + 24\,\psi^5\gamma_e\varphi_i\kappa - 16\,\psi\,\gamma_e l\varphi_e\kappa^3 - \\
-48\,\psi\,\gamma_e{\varphi_e}^2\kappa^3 - -12\,\psi^3\gamma_e l\varphi_e\kappa - 12\,\psi^3\gamma_e{\varphi_e}^2\kappa - 16\,\psi\,\gamma_e l\kappa^3 + \\
+32\,\psi\,\gamma_e\varphi_e\kappa^3 - 12\,\psi^3\gamma_e l\kappa + 24\,\psi^3\gamma_e\varphi_e\kappa - 16\,\psi^6\gamma_i l\varphi_i\kappa^2,
\end{gathered}
\tag{63}
$$

$$
\begin{gathered}
U_2 = 12\,\psi^4{\gamma_i}^2 l{\varphi_i}^2\kappa^4 - 12\,\psi^4{\gamma_i}^2{\varphi_i}^2\kappa^4 - 30\,\psi^4 l{\varphi_i}^2\kappa^4 + \\
+24\,\psi^3\gamma_e\gamma_i l\varphi_e\varphi_i\kappa^3 + 27\,\psi^4 l\varphi_i\kappa^4 + 30\,\psi^4{\varphi_i}^2\kappa^4 - 24\,\psi^3\gamma_e\gamma_i\varphi_e\varphi_i\kappa^3 + \\
+12\,\psi^6 l\varphi_i\kappa^2 + 24\,\psi^6{\varphi_i}^2\kappa^2 + 18\,\psi^4 l\kappa^4 - 27\,\psi^2 l{\varphi_e}^2\kappa^4 - 24\,\psi^2 l\varphi_e\varphi_i\kappa^4 - \\
-3\,\psi^2 l{\varphi_i}^2\kappa^4 - 27\,\psi^4\beta_i\kappa^4 - 54\,\psi^4\varphi_i\kappa^4 + 12\,\psi^6 l\kappa^2 - 3\,\psi^4 l{\varphi_e}^2\kappa^2 - \\
-24\,\psi^4 l\varphi_e\varphi_i\kappa^2 - 18\,\psi^4 l{\varphi_i}^2\kappa^2 + 12\,\psi^2{\gamma_e}^2 l{\varphi_e}^2\kappa^2 - 12\,\psi^6\beta_i\kappa^2 - \\
-24\,\psi^6\varphi_i\kappa^2 + 27\,\psi^2 l\varphi_e\kappa^4 + 12\,\psi^2 l\varphi_i\kappa^4 + 27\,\psi^2{\varphi_e}^2\kappa^4 + \\
+24\,\psi^2\varphi_e\varphi_i\kappa^4 + 3\,\psi^2{\varphi_i}^2\kappa^4 + 12\,\psi^4 l\varphi_e\kappa^2 + 18\,\psi^4 l\varphi_i\kappa^2 + 3\,\psi^4{\varphi_e}^2\kappa^2 + \\
+24\,\psi^4\varphi_e\varphi_i\kappa^2 + 18\,\psi^4{\varphi_i}^2\kappa^2 - 12\,\psi^2{\gamma_e}^2{\varphi_e}^2\kappa^2 + 30\,l\psi^2\kappa^4 - 24\,l{\varphi_e}^2\kappa^4 - \\
-12\,\beta_i\psi^2\kappa^4 - 54\,\psi^2\varphi_e\kappa^4 - 24\,\varphi_i\psi^2\kappa^4 + 30\,\psi^4 l\kappa^2 - 21\,\psi^2 l{\varphi_e}^2\kappa^2 - \\
-18\,\psi^4\beta_i\kappa^2 - 24\,\psi^4\varphi_e\kappa^2 - 36\,\psi^4\varphi_i\kappa^2 + 12\,l\varphi_e\kappa^4 + \\
+24\,{\varphi_e}^2\kappa^4 + 18\,\psi^2 l\varphi_e\kappa^2 + 21\,\psi^2{\varphi_e}^2\kappa^2 + 12\,l\kappa^4 - 24\,\varphi_e\kappa^4 + 18\,\psi^2 l\kappa^2 - \\
-36\,\psi^2\varphi_e\kappa^2 - 24\,\psi^6 l{\varphi_i}^2\kappa^2,
\end{gathered}
\tag{64}
$$

$$
\begin{gathered}
\gamma_\alpha U_1 = -36\,\psi^4\gamma_i l{\varphi_i}^2\kappa^4 + 12\,\psi^4\gamma_i l\varphi_i\kappa^4 + 36\,\psi^4\gamma_i{\varphi_i}^2\kappa^4 - \\
-12\,\psi^2\gamma_i l{\varphi_e}^2\kappa^4 - 24\,\psi^2\gamma_i l\varphi_e\varphi_i\kappa^4 - 12\,\psi^4\beta_i\gamma_i\kappa^4 - 24\,\psi^4\gamma_i\varphi_i\kappa^4 - \\
-24\,\psi^3\gamma_e l\varphi_e\varphi_i\kappa^3 - 12\,\psi^3\gamma_e l{\varphi_i}^2\kappa^3 + 12\,\psi^2\gamma_i l\varphi_e\kappa^4 + 12\,\psi^2\gamma_i{\varphi_e}^2\kappa^4 + \\
+24\,\psi^2\gamma_i\varphi_e\varphi_i\kappa^4 + 12\,\psi^3\gamma_e l\varphi_i\kappa^3 + 24\,\psi^3\gamma_e\varphi_e\varphi_i\kappa^3 + 12\,\psi^3\gamma_e{\varphi_i}^2\kappa^3 + \\
+12\,\psi^2\gamma_i l\kappa^4 - 24\,\psi^2\gamma_i\varphi_e\kappa^4 + 12\,\psi^3\gamma_e l\kappa^3 - 36\,\psi\,\gamma_e l{\varphi_e}^2\kappa^3 - \\
-12\,\psi^3\beta_i\gamma_e\kappa^3 - 24\,\psi^3\gamma_e\varphi_i\kappa^3 + 12\,\psi\,\gamma_e l\varphi_e\kappa^3 + 36\,\psi\,\gamma_e{\varphi_e}^2\kappa^3 + 12\,\psi\,\gamma_e l\kappa^3 - \\
-24\,\psi\,\gamma_e\varphi_e\kappa^3 + 12\,\psi^4\gamma_i l\kappa^4,
\end{gathered}
\tag{65}
$$

$$
\begin{gathered}
U_0 = 18\,\psi^4 l\varphi_i{}^2\kappa^4 - 9\,\psi^4 l\varphi_i\kappa^4 - 18\,\psi^4\varphi_i{}^2\kappa^4 - 9\,\psi^4 l\kappa^4 + 9\,\psi^2 l\varphi_e{}^2\kappa^4 + \\
+18\,\psi^2 l\varphi_e\varphi_i\kappa^4 + 9\,\psi^2 l\varphi_i{}^2\kappa^4 + 9\,\psi^4\beta_i\kappa^4 + 18\,\psi^4\varphi_i\kappa^4 - 9\,\psi^2 l\varphi_e\kappa^4 - \\
-9\,\psi^2 l\varphi_i\kappa^4 - 9\,\psi^2\varphi_e{}^2\kappa^4 - 18\,\psi^2\varphi_e\varphi_i\kappa^4 - 9\,\psi^2\varphi_i{}^2\kappa^4 - 18\,l\psi^2\kappa^4 + \\
+18\,l\varphi_e{}^2\kappa^4 + 9\,\beta_i\psi^2\kappa^4 + 18\,\psi^2\varphi_e\kappa^4 + 18\,\varphi_i\psi^2\kappa^4 - 9\,l\varphi_e\kappa^4 - \\
-18\,\varphi_e{}^2\kappa^4 - 9\,l\kappa^4 + 18\,\varphi_e\kappa^4,
\end{gathered}
\tag{66}
$$

where $\kappa = \sqrt{\Lambda_e}$.

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SHEAR INSTABILITY ANALYSIS OF MHD DISCONTINUITIES IN THE SOLAR WIND CONDITIONS

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We study wave properties and instabilities in a magnetized, anisotropic, collisionless plasma in the fluid approximation using the 16-momentum formalism. In particular, we investigated equations different from the ideal MHD equations by including evolution equations for the heat fluxes with different components along the magnetic field $S_{\parallel}$ and $S_{\perp}$. Main aim of this paper is to study the MHD shear instability that occurs in the contact surface of interaction of the slow (350-400 km/s), fast (600-850 km/s) and CME (900-1200 km/s) components of solar wind, taking into account the anisotropic properties of the wind plasma. A preliminary study of the obtained dispersion equation for solar wind plasma parameters showed that in places of contacts of different streams having the MHD shear instability properties which highly dependent on the plasma anisotropy and heat flow along magnetic field. The results could be used for the interpretation of the observed facts of wave turbulence in corotating interaction regions (CIRs) as well.

Keywords: Magnetohydrodynamics (MHD) – Shear instability – solar wind – 16-momentum transport equation – collisionless plasma

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1. INTRODUCTION

The Magnetohydrodynamics shear instability (MSI) is one of the most important process in astrophysics and geophysics [13], [12]. It can occur in the presence of a two (or more) regions with different physical properties, which are in relative one to another motion. MSI extensively has been theoretically and experimentally studied in such space areas as the solar corona [19], the ionosphere [14], and many other astrophysical situations [8]. The well-known version of MSI is the tangential discontinuity at the interaction region between the solar wind and the Earth's magnetosphere. Chandrasekhar [2] developed linear theory of the MSI for the case when there is sufficient transverse velocity shear rates across the tangential discontinuity. He obtained dispersion relation with important properties [17], [18] for an incompressible homogeneous isotropic plasma on two sides of a discontinuous tangential flow and discussed effect of the homogeneous tangential magnetic field on instability. Until present time, the MSI is under the increasing attention and has been extensively studied both theoretically [24], [1] and experimentally [9] with many applications in the astrophysical context [10], [11].

The problem of the MSI of an anisotropic pressure plasma has been analyzed (e.g. [22]). Investigations of the MSI instability of homogeneous anisotropic pressure plasma have been done by [23]. In [4] there has been carried out the study of the MSI instability of anisotropic pressure plasma in presence of magnetic field using Chew-Goldberger and Low (CGL) approximation. The history of study of the wave instabilities using 16-momentum fluid formalism began with series works of Dzhalilov and Kuznetsov [15], [16], [7], [5], [6]. In paper [7] authors assumed a magnetized, anisotropic, collisionless, rarefied hot plasma and derived the general dispersion relation for linear compressible wave modes. Important properties implied by this dispersion relation removed the discrepancies between the results of the CGL-MHD fluid model and the kinetic theory: i) The mirror slow mode instability criterion in this case was the same as that in the kinetic theory. ii) Similarly, in kinetic studies appear two kinds of fire hose instabilities - incompressible and compressible ones. However, in these studies the effects associated with shear are not considered.

In order to examine the MSI in the tangential discontinuity between collisionless flow regions of different velocities in the solar wind, we use the fluid approximations of the 16-momentum set of the transport equations, following methodology in the paper [7]. As it is known, solar wind can be divided into three components:

- First fast wind component is a major component during the declining and minimum phases of the solar cycle, flows with speeds 600 - 850 kilometers per second. This component outflows from regions with low coronal density

and temperature, which are known as coronal holes. In the coronal holes the magnetic field is strong and the field lines are open to interplanetary medium.

- Second component of the solar wind outflows with low-speed 350 - 400 kilometers per second. Slow wind component has the base as the equatorial coronal streamer belt. In the ecliptic dominate interestings physics where the fast and slow components alternate in the solar wind. Since the Sun rotates, both components outflow from the Sun by well-known trajectory an Archimedean spiral. As it should be the high-speed streams outrun the slow-speed flows whereupon arising co-rotating interaction regions (CIRs). CIRs have enhanced density and magnetic field than surrounding regions and play an important role in solar-terrestrial relations.

- Although in the ascending phase of the solar activity cycle and at solar maximum, the average solar wind speed slows, the solar wind flow basically associated with the coronal mass ejections (CMEs) at varying speeds 900 - 1200 kilometers per second. CMEs erupt from closed magnetic field regions and reach a peak values of the occurrence rate at solar maximum. At this period, due to fast earthward-directed CMEs we register often occurring geomagnetic storms.

We first study analytically properties of MSI in context 16-momentum fluid formalism, unlike previous works where studied this instability within the isotropic hydrodynamics and magneto-hydrodynamics. In this article we will present simple configuration of tangential discontinuity with an possibility to choose some special cases when to parameters of plasmas such as the pressure, density, initial heat fluxes, plasma-beta and anisotropy are given physical mean values. We will discuss the cases when boundaries of solar wind and CME plasmas are shearing and unstable, by including dimensionless parameter h as ratio of flow speeds, which values provide enough condition for velocity shear across the interfaces. The 16 momentum MHD set of equations for anisotropic ion plasma may be used in the form [20], [21].

$$\frac{d\rho}{dt} + \rho \ \mathrm{div}\mathbf{v} = 0\,, \tag{1}$$

$$\begin{aligned} \rho\frac{d\mathbf{v}}{dt} + \nabla(p_\perp + \frac{B^2}{8\pi}) - \frac{1}{4\pi}(\mathbf{B}\cdot\nabla)\mathbf{B} = \\ = (p_\perp - p_\parallel)\big[\mathbf{h_B}\mathrm{div}\mathbf{h_B} + (\mathbf{h_B}\cdot\nabla)\mathbf{h_B}\big] + \mathbf{h_B}(\mathbf{h_B}\cdot\nabla)(p_\perp - p_\parallel) + \rho\mathbf{g} \end{aligned} \tag{2}$$

$$\frac{d}{dt}\frac{p_\parallel B^2}{\rho^3} = -\frac{B^2}{\rho^3}\Big[B(\mathbf{h_B}\cdot\nabla)\Big(\frac{S_\parallel}{B}\Big) + \frac{2S_\perp}{B}(\mathbf{h_B}\cdot\nabla)B\Big], \tag{3}$$

$$\frac{d}{dt}\frac{p_\perp}{\rho B} = -\frac{B}{\rho}(\mathbf{h_B}\cdot\nabla)\Big(\frac{S_\perp}{B^2}\Big), \tag{4}$$

$$\frac{d}{dt}\frac{S_\parallel B^3}{\rho^4} = -\frac{3p_\parallel B^3}{\rho^4}(\mathbf{h_B}\cdot\nabla)\Big(\frac{p_\parallel}{\rho}\Big), \tag{5}$$

$$\frac{d}{dt}\frac{S_\perp}{\rho^2} = -\frac{p_\parallel}{\rho^2}\Big[(\mathbf{h_B}\cdot\nabla)\Big(\frac{p_\perp}{\rho}\Big) + \frac{p_\perp}{\rho}\frac{p_\perp - p_\parallel}{p_\parallel B}(\mathbf{h_B}\cdot\nabla)B\Big], \tag{6}$$

$$\frac{d\mathbf{B}}{dt} + \mathbf{B}\mathrm{div}\mathbf{v} - (\mathbf{B}\cdot\nabla)\mathbf{v} = 0, \tag{7}$$

$$\mathrm{div}\mathbf{B} = 0. \tag{8}$$

where $\mathbf{h_B} = \mathbf{B}/B$ and $d/dt = \partial/\partial t + \big(\mathbf{v}\cdot\nabla\big)$ is the convective derivative. Here $S_\parallel$ and $S_\perp$ are the heat fluxes along the magnetic field due to parallel and perpendicular thermal kinetic motions of ions. If the heat fluxes are neglected, $S_\parallel = 0$ and $S_\perp = 0$ we obtain (1-4),(7) and (8) closed system of equation which called the CGL(Chew-Goldberger-Low) equations, see the pioneering work by [3]. Given (1-8) 16-moments set of equations, one can consider the heat fluxes more completely form compared to the CGL equations.

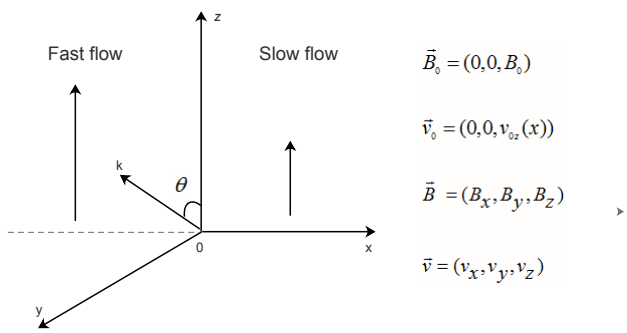

Fig. 1. Geometry in 3-D MHD configuration.

We consider, geometry of the MHD plasma flow, as in the fig. 1. The equilibrium velocity $\mathbf{v}_0$ has a component only in z-direction, which varies on x-axis, $\mathbf{v}_0 = (0, 0, \mathrm{v}_{0z}(x))$ and the background magnetic field $\mathbf{B}_0$ directed along z-axis. We assume that the background state with non-zero thermal fluxes is homogeneous (the gravitational acceleration, $g = 0$ and the following quantities $\rho_0, p_{\perp 0}, p_{\parallel 0}, B_0, S_{\perp 0}, S_{\parallel 0}$ are constant). We considered the stability of the system to linear perturbations of all the physical variables, according to the form $f = f_0 + f'(x, y, z, t)$, where $f'(x, y, z, t) \sim F(x)\ exp[i(k_y y + k_z z - \omega t)]$. Here, ω is the wave frequency and k_y, k_z are the wave numbers. After linearization of equations (1-8) we obtain

$$\frac{\rho^{'}}{\rho_0} = \frac{k_y}{\omega_z}\mathrm{v}_y + \frac{k_z}{\omega_z}\mathrm{v}_z - \frac{i}{\omega_z}\frac{\partial \mathrm{v}_x}{\partial x}\,, \tag{9}$$

$$\rho_0\omega_z\mathrm{v}_x + i\frac{\partial p^{'}{}_{\perp}}{\partial x} + i\frac{B_0}{4\pi}\frac{\partial B_z}{\partial x} + \frac{B_0}{4\pi}k_z B_x - \Delta k_z\frac{B_x}{B_0} = 0\,, \tag{10}$$

$$\rho_0\omega_z\mathrm{v}_y - k_y(p^{'}{}_{\perp} + \frac{B_0}{4\pi}B_z) + \frac{B_0}{4\pi}k_z B_y - \Delta k_z\frac{B_y}{B_0} = 0\,, \tag{11}$$

$$\rho_0\omega_z\mathrm{v}_z + i\rho_0 V_0'(x)\mathrm{v}_x + i\frac{P}{B_0}\frac{\partial B_x}{\partial x} - \frac{\delta}{B_0}k_y B_y - k_z p^{'}{}_{\parallel} = 0\,, \tag{12}$$

$$p^{'}{}_{\perp} = p_{\perp 0}\frac{a_1(x)}{a_0(x)}\frac{B_z}{B_0} + p_{\perp 0}\frac{a_2(x)}{a_0(x)}\frac{\rho^{'}}{\rho_0}\,, \tag{13}$$

$$p^{'}{}_{\parallel} = p_{\parallel 0}\frac{b_1(x)}{b_0(x)}\frac{B_z}{B_0} + p_{\parallel 0}\frac{b_2(x)}{b_0(x)}\frac{\rho^{'}}{\rho_0}\,, \tag{14}$$

$$\omega_z B_x + B_0 k_z \mathrm{v}_x = 0\,, \tag{15}$$

$$\omega_z B_y + B_0 k_z \mathrm{v}_y = 0\,, \tag{16}$$

$$\frac{\partial B_x}{\partial x} + ik_y B_y + ik_z B_z = 0\,. \tag{17}$$

Here, $P = p_{\parallel 0} - p_{\perp 0}$, $V_0'(x) = \partial \mathrm{v}_0/\partial x$ and $\omega_z = \omega - k_z\ V_0(x)$ is the Doppler shifted frequency. We can easily find expression of v_x , v_y and B_y from (15), (16) and (17) respectively and by using them into (9) and (13)-(14) we obtain following

$$\frac{\rho^{'}}{\rho_0} = -\frac{i\ V_0'(x)}{\omega_z\ B_0}B_x + \frac{B_z}{B_0} + \frac{k_z}{\omega_z}\ \mathrm{v}_z\,, \tag{18}$$

$$p'_{\perp} = \frac{p_{\perp 0}}{B_0}\Big(-\frac{iV_0'(x)}{\omega_z}\frac{a_2}{a_0}B_x + \frac{a_1 + a_2}{a_0}B_z + \frac{a_2}{a_0}\frac{k_z\ B_0}{\omega_z}\mathrm{v}_z\Big)\,, \tag{19}$$

$$p'_{\parallel} = \frac{p_{\parallel 0}}{B_0}\Big(-\frac{iV_0'(x)}{\omega_z}\frac{b_2}{b_0}B_x + \frac{b_1 + b_2}{b_0}B_z + \frac{b_2}{b_0}\frac{k_z\ B_0}{\omega_z}\mathrm{v}_z\Big)\,, \tag{20}$$

after inserting of these relations into (10-12) we obtained that equations depend only on B_x, B_z and v_z

$$\beta_A k_z B_x + \frac{\partial}{\partial x}\left(\alpha\frac{V_0'(x)\ a_2}{\omega_z\ a_0}B_x + i\ \beta_0 B_z + i\ B_0\alpha k_z\frac{a_2}{a_0\omega_z}\mathrm{v}_z\right) = 0\,, \tag{21}$$

$$\frac{ik_y\alpha V_0'(x)}{\omega_z}\frac{a_2}{a_0}B_x + \frac{i\beta_A k_z}{k_y}\frac{\partial B_x}{\partial x} - \frac{k_y^2\beta_0 + k_z^2\beta_A}{k_y}B_z - \frac{\alpha k_z k_y B_0}{\omega_z}\frac{a_2}{a_0}\mathrm{v}_z = 0\,, \tag{22}$$

$$\frac{k_z^2}{\omega_z}\mathrm{v}_z - \frac{ik_z V_0'(x)}{B_0\omega_z}B_x + \frac{\beta_1 k_z}{B_0}B_z = 0\,. \tag{23}$$

Firstly, excluding v_z from (23) and insert into (22) then we get expressions for both variables v_z and B_z in terms of B_x

$$\mathrm{v}_z = \frac{i\ V_0'(x)}{k_z\ B_0} B_x - \frac{\omega_z\ \beta_1}{k_z\ B_0} B_z\,, \tag{24}$$

$$B_z = \frac{i\ k_z \beta_A}{k_y^2 \beta_\star + k_z^2 \beta_A} \frac{\partial B_x}{\partial x}\,, \tag{25}$$

substituting the last two expressions into (21) we obtain second-order differential equation of the form

$$\frac{\partial}{\partial x}\Big(A(x)\ \frac{\partial B_x}{\partial x}\Big) - \beta_A(x)\ B_x = 0\,, \tag{26}$$

where

$$A(x) = \frac{\beta_A \beta_\star}{k_y^2 \beta_\star + k_z^2 \beta_A}\,, \tag{27}$$

$$\begin{gathered} \beta_A = \beta - \bar{\alpha} - \frac{1}{\eta^2}\,, \quad \beta_\star = \beta_0 - \alpha \frac{a_2}{a_0} \beta_1 \\ \beta_0 = \alpha \frac{a_1 + a_2}{a_0} + \beta\,, \quad \beta_1 = \frac{b_0 \bar{\alpha} - b_1 - b_2}{b_0/\eta^2 - b_2}\,, \end{gathered} \tag{28}$$

coefficients $a_{0,1,2}$ and $b_{0,1,2}$ are determined as

$$\begin{gathered} a_0 = 1 - \eta^2\,, \quad a_1 = 1 - 2\gamma\eta - \bar{\alpha}\eta^2\,, \quad a_2 = 1 + 2\gamma\eta - \eta^2 \\ b_0 = 1 - 3\eta^2\,,\ b_1 = 2\gamma\eta(\alpha - 2) - 2\,,\ b_2 = 3 + 4\gamma\eta - 3\eta^2\,, \end{gathered} \tag{29}$$

where for the basic unperturbed physical parameters(indexes zero are dropped) were used dimensionless parameters

$$\begin{gathered} \alpha = \frac{p_\perp}{p_\parallel}\,, \overline{\alpha} = 1 - \alpha\,, \beta = \frac{B^2}{4\pi p_\parallel} = \frac{\mathrm{v}^2{}_A}{c^2{}_\parallel}\,, c^2{}_\parallel = \frac{p_\parallel}{\rho} \\ \eta = \frac{c_\parallel k_z}{\omega_z}\,, \gamma_\parallel = \frac{S_\parallel}{p_\parallel c_\parallel}\,, \gamma_\perp = \frac{S_\perp}{p_\perp\, c_\parallel}\,, \gamma = \gamma_\parallel = \gamma_\perp\,. \end{gathered} \tag{30}$$

The $\gamma = \gamma_\parallel = \gamma_\perp$ is a simplified approach (see [7]). Here $c_\parallel$ is parallel sound speed, θ wave propagation angle relative to magnetic field, α anisotropy parameter, γ background heat fluxes parameter, β inversely proportional to the plasma beta ($\beta = 2/\beta_{pl}$). We also assume for $k_z = k\cos\theta$, $k_y = k\sin\theta$ and after denoting $\cos^2\theta \equiv \ell$ we can write $k_z = k\sqrt{\ell}$ and $k_y = k\sqrt{1-\ell}$. Imposing substitution $\xi = 1/\eta$ and derivative with respect to x

$$\frac{\partial}{\partial x} = -\frac{V_0'(x)}{c_\parallel} \frac{\partial}{\partial \xi}\,,$$

into (26), it yields

$$\frac{V_0'(x)}{c_{\parallel}^2}\frac{\partial}{\partial\xi}\Big(A(\xi)V_0'(x)\frac{\partial B_x}{\partial\xi}\Big)-\beta_A(\xi)B_x=0\,, \tag{31}$$

and accordingly β_A and $\beta_\star$ can be written as follows

$$\beta_A=\beta-\bar{\alpha}-\xi^2\,, \tag{32}$$

$$\beta_\star=\beta+2\,\alpha+2\,\alpha^2\,\frac{\xi^4+2\,\gamma\,\xi^3+2\,\gamma^2\xi^2-5\,\xi^2-6\,\gamma\,\xi+3}{(\xi^4-6\,\xi^2-4\,\gamma\,\xi+3)\,(\xi^2-1)}\,. \tag{33}$$

The coefficients of the equation (31) are variable and they are complex functions of ξ. Furthermore, the velocity profile $V_0(x)$ is still arbitrary function of x - coordinates. In case when two plasma flows meet, there is a transition layer with a finite thickness between them. The thickness of the transition layer is determined by the flow velocities and physical parameters in each flow. In the simplest case, to simulate this situation we take a profile as

$$V_0(x)=\frac{V_{02}e^{\sigma x}+V_{01}e^{-\sigma x}}{e^{\sigma x}+e^{-\sigma x}}\,,\quad \sigma>0 \tag{34}$$

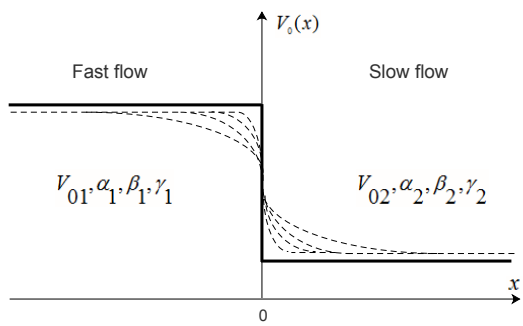

Fig. 2. Schematic representation of MHD plasma flow. Different line profiles correspond to different values of σ in (31).

Here, $L=1/\sigma$ parameter characterizes the thicknesses of the transition layer. Fig. 2 shows schematically the various profiles of $V_0(x)$ for different values of σ. With increase of σ, the transition layer thickness decreases sharply, and eventually there is a discontinuity (solid line in fig. 2). We use new variables

$$u=\frac{V_0(x)}{V_{02}}=\frac{1+he^{-2\sigma x}}{1+e^{-2\sigma x}}\,,\qquad h=\frac{V_{01}}{V_{02}}\,,$$

$$\eta=\frac{1}{M\Big(\Omega-u\Big)}\,,\quad M=\frac{V_{02}}{c_{\parallel}}\,,\ \Omega=\frac{\omega}{k_zV_{02}}\,,$$

$$\frac{\partial}{\partial x} = 2\sigma \, \frac{(u-1)(u-h)}{h-1} \frac{d}{du}.$$

We get second order differential equation with 2 singularity points $u = 1$ and $u = h$

$$q_1(u)(u-1)^2(u-h)^2 y''(u) + q_2(u)(u-1)(u-h)y'(u) + q_3(u)y(u) = 0\,. \quad (35)$$

The resulting new equation has a regular singularity at points $V_0 = V_1$ and $V_0 = V_2$. This equation is a hypergeometric type. Coefficients $q_1(u)$, $q_2(u)$ and $q_3(u)$ respectively are defined as

$$q_1(u) = A(u)\,,$$

$$q_2(u) = \Big[(u-1)(u-h)\frac{dA(u)}{du} + 2\Big(u - \frac{1+h}{2}\Big)A(u)\Big]\,,$$

$$q_3(u) = -\varepsilon\ \beta_A(u)\,,$$

where

$$A(u) = \frac{\beta_A \beta_\star}{(1-\ell)\beta_\star + \ell\beta_A}\,, \quad \varepsilon = \frac{k^2(h-1)^2}{4\sigma^2}\,.$$

With long ($\varepsilon \to 0$) and short ($\varepsilon \to \infty$) wavelengths compared to thickness of the transition layer, equation (35) can be solved by asymptotic methods. In this paper, we will be limited to the case of a profile in the form of a tangential discontinuity.

2. THE GENERAL CONDITION OF SHEAR INSTABILITY

According to the paper [7], it is known that firehose and mirror instabilities modified with heat fluxes appear in anisotropic homogeneous plasma. Furthermore, there may be other types of instabilities, directly related to the heat flux along the magnetic field. Our purpose is to explore the shear instability in the anisotropic medium. Therefore, first we should consider the properties of this instability disregarding the effects due to heat flux. From this moment on we assume that $\gamma = 0$ throughout the following text. Multipling both terms of the equation (26) to conjugate of B_x and integrating from negative to positive infinity we get

$$\int\limits_{-\infty}^{+\infty} \frac{d}{dx}\left(\beta_\star \frac{dB_x}{dx}\right) B_x^\star \, dx - \int\limits_{-\infty}^{+\infty} \beta_A \left|B_x\right|^2 dx = 0\,. \quad (36)$$

After partial integration with boundary conditions $B_x\Big|_{x=\pm\infty} = 0$ we obtain

$$\int\limits_{-\infty}^{+\infty} \beta_\star \left|B_x'\right|^2 dx + \int\limits_{-\infty}^{+\infty} \beta_A \left|B_x\right|^2 dx = 0\,. \tag{37}$$

Taking the limit $\gamma \to 0$ and by introducing new variable $t = \xi^2$ in (32-33) yields

$$\beta_A = \beta - \bar{\alpha} - t\,, \tag{38}$$

$$\beta_\star = \beta + 2\,\alpha + \frac{\alpha^2}{2}\left(\frac{2}{t-1} + \frac{1}{t-3-\sqrt{6}} + \frac{1}{t-3+\sqrt{6}}\right). \tag{39}$$

For plasma instability with background flow, we must notice that, the frequency ω and the variable $t = \xi^2$ are complex, so we can split them into real and imaginary parts,

$$\omega = \omega_r + i\,\omega_i\,,$$

$$t = \xi^2 = \frac{(\omega_r - k_z V_0(x))^2 - \omega_i^2}{k_z^2 c_\parallel^2} + i\,\frac{2\omega_i(\omega_r - k_z V_0(x))}{k_z^2 c_\parallel^2} = t_r + i\,t_i\,.$$

Let $\mathrm{v}_0(x) = V_0(x)/c_\parallel$- normalized velocity, $\mathrm{v}_{ph} = \omega_r/k_z c_\parallel$- normalized phase velocity of waves, $\Gamma = \omega_i/k_z c_\parallel$ - increment of instability, $\Lambda = \mathrm{v}_{ph} - \mathrm{v}_0(x)$, so $t_r = \Lambda^2 - \Gamma^2$, $t_i = 2\Gamma\Lambda$. Then the imaginary part of (37) is reduced to

$$\int\limits_{-\infty}^{+\infty} Q(x)\Lambda(x)\,dx \;=\; 0\,, \tag{40}$$

where

$$Q(x) = \frac{\alpha^2}{2}\left(\frac{2}{\left|t-1\right|^2} + \frac{1}{\left|t-3-\sqrt{6}\right|^2} + \frac{1}{\left|t-3+\sqrt{6}\right|^2}\right)\left|B_x'\right|^2 + \left|B_x\right|^2.$$

As $Q(x) > 0$, then for the fulfilling of (40) it is necessary that the sign of difference $\Lambda = \mathrm{v}_{ph} - \mathrm{v}_0(x)$ must be changed. That means to develope the instability ($\omega_i > 0$) the velocities should satisfy inequality $\mathrm{v}_{02} < \mathrm{v}_{ph} < \mathrm{v}_{01}$ this is the condition of MSI onset. Here $\mathrm{v}_{01} = \mathrm{v}_0(x)\Big|_{x\to-\infty}$, $\mathrm{v}_{02} = \mathrm{v}_0(x)\Big|_{x\to+\infty}$. The real part of (37)

$$\int\limits_{-\infty}^{+\infty} \mathrm{Re}(\beta_\star)\left|B_x'\right|^2 dx + \int\limits_{-\infty}^{+\infty} (\beta + \alpha - 1 - t_r)\left|B_x\right|^2 dx = 0\,, \tag{41}$$

(41) is fulfilled if the condition of fire-hose instability $\beta + \alpha < 1$ is satisfied, where

$$\mathrm{Re}(\beta_\star) = \beta + 2\,\alpha + \alpha^2\left(\frac{t_r - 1}{|t-1|^2} + \frac{t_r - 3 - \sqrt{6}}{2|t-3-\sqrt{6}|^2} + \frac{t_r - 3 + \sqrt{6}}{2|t-3+\sqrt{6}|^2}\right).$$

3. STEP DISCONTINUITY VELOCITY PROFILE

In this paper for simplicity we are assuming a step discontinuity function($\sigma \to \infty$) for the velocity profile $\begin{cases} V_{01} & x < 0 \\ V_{02} & x > 0 \end{cases}$ as shown in Fig. 2(solid line). Substituting this velocity profile into (26) we can obtain equation with constant coefficients which can be easily solved for each flow, separately. This yield the following solution

$$B_x = C_1\, e^{-k_x\, x}, \quad x > 0 \qquad B_x = C_2\, e^{k_x\, x}, \quad x < 0 \tag{42}$$

here

$$k_x = \sqrt{-\frac{k_y^2\beta_\star + k_z^2\beta_A}{\beta_\star}}\,, \qquad Re(k_x) > 0 \tag{43}$$

and all variables can be written in terms of B_x as follows

$$\mathrm{v}_x = -\frac{\omega_z}{B_0 k_z} B_x\,, \tag{44}$$

$$\mathrm{v}_y = -\frac{\omega_z}{B_0 k_z}\,\frac{i\ k_y\beta_\star}{k_y^2\beta_\star + k_z^2\beta_A}\frac{\partial B_x}{\partial x}\,, \tag{45}$$

$$\mathrm{v}_z = -\frac{\omega_z}{B_0}\,\frac{i\ A\beta_1}{\beta_\star}\frac{\partial B_x}{\partial x}\,, \tag{46}$$

$$B_y = \frac{i\ k_y A}{\beta_A}\frac{\partial B_x}{\partial x}\,, \tag{47}$$

$$B_z = \frac{i\ k_z A}{\beta_\star}\frac{\partial B_x}{\partial x}\,, \tag{48}$$

$$\rho' = \frac{\rho_0}{B_0}\frac{i\ k_z A(1-\beta_1)}{\beta_\star}\frac{\partial B_x}{\partial x}\,, \tag{49}$$

$$p'_\perp = \frac{a_1 + (1-\beta_1)a_2}{a_0}\frac{p_{\perp 0}}{B_0}\frac{i\ k_z A}{\beta_\star}\frac{\partial B_x}{\partial x}\,, \tag{50}$$

$$p'_\parallel = \frac{b_1 + (1-\beta_1)b_2}{b_0}\frac{p_{\parallel 0}}{B_0}\frac{i\ k_z A}{\beta_\star}\frac{\partial B_x}{\partial x}\,. \tag{51}$$

We should require that the found solution must satisfy the boundary conditions

$$\left\{\rho_0 \mathrm{v}_x\right\}_{x=0} = 0 \quad (\text{mass flux})\,, \tag{52}$$

$$\left\{p'_\perp + \frac{B_0 B_z}{4\pi}\right\}_{x=0} = 0 \quad (\text{total pressure})\,. \tag{53}$$

The braces in the latter two boundary conditions indicate that the terms inside those brackets represent differences of parameters for the two different flows. For simplicity we use average of velocities($\overline{V} = (V_{0_1} + V_{0_2})/2$) to make expression for η and ω_z dimensionless, which can be written

$$\omega_z = k_z \overline{V}(V_F - m)\,, \quad \eta = \frac{c_{\|}}{V_f - V_0} = \frac{1}{M(V_F - m)} = \frac{1}{\xi}\,, \tag{54}$$

here

$$V_F = \frac{V_f}{\overline{V}}, \quad V_f = \frac{\omega}{k_z}\,, \quad M = \frac{\overline{V}}{c_{\|}}, \quad m = \frac{V_0}{\overline{V}}. \tag{55}$$

Applying condition (52) and considering expression for v_x from (15)

$$\left\{ \rho_0 \frac{\omega_z}{k_z\, B_0} B_x \right\}_{x=0} = 0\,,$$

or

$$\frac{\rho_{0_1}\, \omega_{z1}}{B_{01}}\, C_1 = \frac{\rho_{0_2}\, \omega_{z2}}{B_{02}}\, C_2\,. \tag{56}$$

from last expression it follows that

$$\frac{C_1}{C_2} = \frac{\rho_{02}}{\rho_{01}} \frac{B_{01}}{B_{02}} \frac{\omega_{z2}}{\omega_{z1}},$$

and expression for condition (53)

$$\left\{ \left(\frac{\beta_\star - \beta}{\alpha} + \frac{\beta}{\alpha} \right) \frac{p_{\perp_0}}{B_0} \frac{i\; k_z \beta_A}{k_y^2 \beta_\star + k_z^2 \beta_A} \frac{\partial B_x}{\partial x} \right\}_{x=0} = 0\,. \tag{57}$$

Simplifying and regarding to B_x component (57) can be written also as follow

$$\sum_{i=1}^{2} \frac{\beta_{A_i}\, c_{\|_i}}{\xi_i\, \kappa_i} = 0, \qquad \kappa = \sqrt{1 - \ell + \ell\, \frac{\beta_A}{\beta_\star}}, \tag{58}$$

where $i = 1, 2$ indicate 2 different flows. We assume that after squaring the expression (58) can be written as ratio of two polynoms ($N(\xi)/D(\xi)$), and for different flows we can write dispersion relation as

$$N_1(\xi_1)\, D_2(\xi_2) + N_2(\xi_2)\, D_1(\xi_1) = 0, \tag{59}$$

here

$$\xi_1 = X - \Delta, \quad \xi_2 = X + \Delta, \tag{60}$$

where $X = M\,(V_F - 1)$ and $\Delta = M\,(h-1)/(h+1)$. Taking $\gamma = 0$ let the equation (59) pass to the polynom with even degrees of X. Which also allow us to get polynomial equation of 8 order for $f = X^2$

$$\sum_{n=1}^{8}(a_n - \ell\, b_n)f^n + (\Delta^2 - \alpha - \beta + 1)(a_0 - \ell\, b_0) = 0\,. \tag{61}$$

Due to a large expressions of a_n and $b_n(n = 0..8)$ we do not include these coefficients explicitly in this paper. The equation has been solved numerically. In (61) C_0 specify threshold condition. In case $f = 0$ we can get a critical expression for $\ell_c = a_0/b_0$. It should be noted that, in comparison with (58), the dispersion equation (61) contains additional false roots. These roots must be excluded by checking (58).

4. LIMITING CASE

Let us consider the case of perpendicular propagation ($k_z \to 0$). First we take all parameters same except velocity, $V_{01} \neq V_{02}$ and assuming $\ell \to 0$ which implies $\kappa \to 1$. Hence it leads that

$$\sum_{i=1}^{2}\frac{\alpha + \beta - 1 - {\xi_i}^2}{\xi_i} = 0. \tag{62}$$

Due to (60), last equation (62) can be written as

$$\frac{\alpha + \beta - 1 - (X - \Delta)^2}{X - \Delta} + \frac{\alpha + \beta - 1 - (X + \Delta)^2}{X + \Delta} = 0.$$

Finally, we obtain solutions as follows

$$X = \pm\sqrt{\Delta^2 + \beta + \alpha - 1}. \tag{63}$$

For MSI condition $\Delta^2 + \beta + \alpha < 1$ should satisfy. This means that the velocity shift suppresses the classical fire-hose instability.

5. SHEAR FLOWS WITH SAME PHYSICAL PARAMETERS

In this paper our main goal is to calculate the growth rate of MSI in an anisotropic medium. Therefore, to avoid other plasma instabilities, we assume in addition to $\gamma = 0$, also condition of absence of firehose ($\alpha + \beta < 1$) and mirror instabilities($2\alpha + \beta < 2\alpha^2$). Let's consider the case when all the parameters of

the anisotropic plasma of both shear flows are the same. Only the flow velocities differ, $V_{01} \neq V_{02}$. It is well known that the firehose instability occurs if the condition $\alpha + \beta < 1$, or $p_{\|} > p_{\perp} + 2p_{mag}$ is observed. This is possible only if $\alpha < 1$. The condition of mirror instability occurrence is more complex. However, during quasi-transverse propagation ($\ell \to 0$), it is simplified and takes the form of $2\alpha + \beta < 2\alpha^2$. For pressures $p_{\perp}^2 > p_{\|}(p_{\perp} + p_{mag})$. It is clear to observe this condition one should have $\alpha > 1$. We chose for supersonic flows ($M > 1$) such parameters α and β, to avoid the firehose and mirror conditions. Solution of the dispersion equation (61) for this example are shown in Fig. 3. Fig. 3 shows the phase velocities of waves along the flow, normalized by the average flow velocity, $Re(V_F) = Re(\omega/k_z\bar{V})$.

As expected, if MSI occurs, then the real parts of the solution should be in the range $v_{02} < v_{ph} < v_{01}$. In our notations, taking into account that $v_{ph} = M/\sqrt{\ell}\, Re(V_F)$, this corresponds to the condition of $m_2/\sqrt{\ell} < Re(V_F) < m_1/\sqrt{\ell}$. Here $m_1 = V_{01}/\bar{V} = 2h/(1+h)$, $m_2 = V_{02}/\bar{V} = 2/(1+h)$. As follows from the figures, the fusion of modes occurs between the bifurcation points (branching solutions). This causes instability. The number of bifurcation points increases with growth of ℓ. In quasi-longitudinal propagation of waves of $\ell \to 1$, along with MSI the oblique firehose instability occurs as well. MSI increment increases with growth of propagation angle.

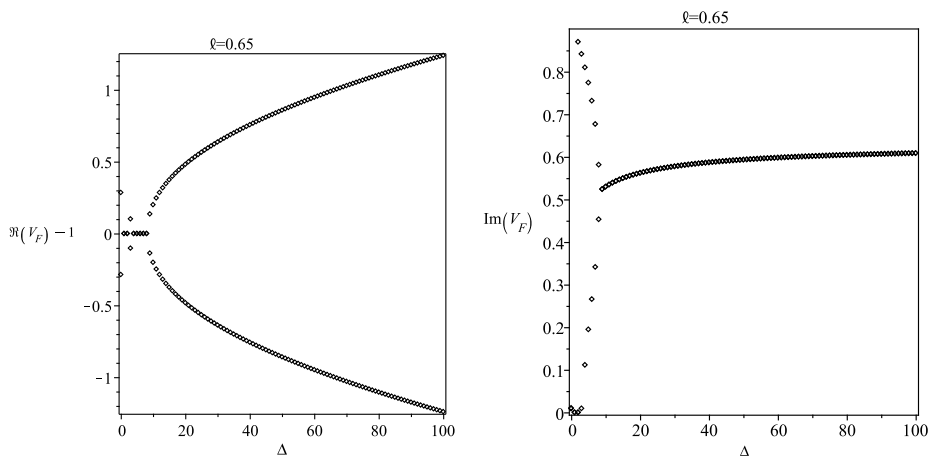

Fig. 3. The normalized phase velocity V_F of waves depends on ratio of shear velocities Δ for given $\ell = cos^2\theta$ values. Here anisotropy parameters $\alpha_1 = \alpha_2 = 1.4$, the inverse plasma beta $\beta_1 = \beta_2 = 3$ and the Mach number $M_1 = M_2 = 8$.

The most unexpected result here is the behavior of the MSI increment. In a classic isotropic case, MSI increment is defined as $|h-1|/(h+1)$, i.e. with the growth of h, it tends to one. In the anisotropic plasma the increment has maximum and MSI occurs within a certain range of values of h. This limit is much narrower with an increase of ℓ. In this case plasma instability occur, however MSI dominates. Another interesting fact is that, slow modes (respect to the average of velocities, $V_F < \overline{V}$), $Re(V_F) < 1$ with the increasing of h change their direction of propagation (reflection of modes on velocity discontinuity).

6. DISCUSSIONS AND CONCLUSIONS

We studied shear plasma flows along the magnetic field in the context 16−momentum MHD equations neglecting gravity effects. For an arbitrary flow longitudinal velocity profile with transverse gradient, considered linear wave perturbations, which are propagating in an arbitrary direction. In the presence of an anisotropic heat fluxes it has been assumed, that both flows are anisotropic and homogeneous. The interfacial layer thickness is finite because of continuous shear velocity profile. For that case we obtained second order differential equation in general form. The special case considered in more detail, when interfacial layer has a zero thickness, i.e. between of the plasma flows presented tangential discontinuity. For such case, resulting equation becomes easily solvable.

As next step applying the boundary conditions on the discontinuity, we obtained the dispersion relation in general form, which can be presented as a polynomial equation of 16th degree (61). As is known [e.g., see. [7]], the wave modes are described by the equation with 8th degree in each environment separately. At the interface of the two plasma flow regions occur KH wave modes, which are described by the 2nd order equation. The phase velocity of these modes along with the flow is equal to the average velocity of the flow, $Re(V_F) = 1$. Thus, the shear flow is "associated" to all of these modes and the interaction of all wave modes is described by the 16th degree equation.

In this paper we restricted ourselves to the study of MSI in an anisotropic medium. Eliminating the heat flux, and selecting the same physical parameters in both flows, we solved the dispersion equation. The task parameters for supersonic flows were chosen so to filter out classic firehose and mirror (during quasi-transverse propagation) plasma instabilities. It turned out that MSI increment, unlike the usual hyrodynamic case, has maximum, and the instability has two thresholds in the anisotropic case. In other words, MSI occurs within $h_1 < h < h_2$, where $h = V_{01}/V_{02}$ characterizes the difference between the flow velocities. The results obtained can be used to explain the observed wave turbulence in the solar wind, the winds of the stars and the planetary magnetospheres.

The side contact discontinuities are possible in CIR, which occur in the interaction of fast and slow components of the solar wind. There is always turbulence in the solar wind, of any size, over time and space. However energy is mainly concentrated in the large scales, which are much large than the cyclotron radius and period, respectively. Therefore, the observed turbulence can be described in the MHD approximation. The observed turbulence has the characteristic velocity (10-50 km/s) much smaller than the flow velocity. There is a debate about the cascade mechanism of dissipation of large scale turbulence energy. This can occur if the energy of such large scales turbulence is transferred to small scales and they reach kinetic dissipation levels. For this process development of strong instabilities plays important role. MSI is one of the most rapidly growing instabilities among others.

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DETERMINATION OF THE PLANET SATELLITE EXTERNAL GRAVITATIONAL FIELD PARAMETERS BY APPROXIMATION METHOD

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Decomposition of the gravitational potential of planet satellites in a Laplace series with a large number of series members is a challenge, and the series does not converge when approaching the body surface, thus causing a need to improve the computing technique. The purpose of this paper is to determine the gravitational potential of satellites, which we classify as planetoids. The physical properties of the satellites were analyzed, and the parameters of their gravitational fields were determined by approximation method.

Keywords: Planet satellites – Figure – Gravitational field – Form – Mass

1. INTRODUCTION

Modern ground and space observations allow obtaining the physical and dynamic characteristics of the satellites, but, in turn, their verification requires constructing the mathematical models, internal structure clarification, orbital libration, physical libration and, definitely, the parameters of internal and external gravitational fields.

Search for various forms of solutions under different assumptions about mass and its distribution continues in celestial mechanics all the time [11]. We are interested in the parameters of gravitational field of planet satellites, which we can determine by approximation methods.

We have two main promlems when determining the potential: direct and inverse, like in conventional geodetic practice. Thus the analytical description of the direct problem potential is a solution of the first problem. The solution of the inverse problem is to establish the surface of the celestial body, which could

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express its potential and allowed to present it easier than as a series of spherical functions [12].

2. ANALYSIS OF STUDIES AND PUBLICATIONS

A large number of planet satellites have significant compression due to gravitational interaction with their planets, i.e. their gravitational potential should be described by an ellipsoid. However, nowadays there are a number of issues related to establishment and operation of mathematical models of dynamic figures and gravitational fields [8, 13]. Various methods are used for modeling the gravitational field, such as decomposition in a Laplace's series, decomposition in Lame's function, point masses method, equigravitational figures and other methods [4,5,14].

As for the theory of figures, the most important methods were proposed by P. Laplace, J. Lagrange, C. McLaurin, K. Jacobi, P. Pitsetti A. Clairaut, A. Legendre, C. Gauss, A. Lyapunov, A. Poincare, M. Subbotin, D. Darwin, D. Jeans, J. Liouville et al. These issues are also described in papers of scholars. such as K. Kholshevnikov, A. Orlov, V. Panteleev, V. Antonov, O. Zheleznyak, B. Kondratiev, N. Hrushynskyi, A. Konopliv, Y. Barkin, O. Kuskov, G. Jeffreys, K. Murray, C. Dermott, J. Anderson, R. Jacobson et al.

3. MAIN MATERIALS

Suppose the planetary satellite parameters are above critical as for self-gravitating structures [3, 7], and since the planet satellites rotate around their axes of rotation, their shape will not be a regular sphere, but will feature some polar compression. The gravitational fields and planetary satellite figures shall be determined by distribution of density inside their bowels, angular velocity and the influence of their planets. We assume that the planet satellite is rotating synchronously about the circular orbit in the equatorial plane of the planet.

It is obvious that there is a relationship between the figure and gravitational field parameters. When the field parameters are known, one can determine the figure parameters and vice versa. Suppose there is a random satellite of the planet with axes a - aimed at this planet, b - aimed to the direction of the orbital motion, and c - between the poles (Fig. 1).

The shape of the satellite is described by triaxial inertial ellipsoid with the major semi-axes $a > b > c$ and the main central inertia moments $I_{xx} < I_{yy} < I_{zz}$. Suppose Φ is a potential of all forces affecting the satellite, i.e. the constant potential surface with the constant pressure and density. Since this surface is the

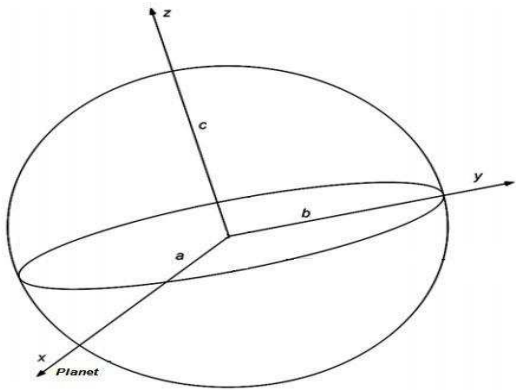

Fig. 1. Triaxial satellite ellipsoid.

surface of constant potential, then $\Phi = const$. It is clear that, in general, at each point of the field we have some potential value, and it may vary from point to point.

According to the classic representation following Laplace-Legendre method, the external gravitational potential of an isolated body with mass M and volume V can be represented as follows [1]:

$$\Phi(r,\theta,\varphi) = \frac{GM}{r}\left\{1 - \sum_{n=0}^{\infty} J_n \left(\frac{R}{r}\right)^n P_n(\cos\theta) + \right. \\ \left. + \sum_{n=0}^{\infty}\sum_{k=1}^{n} \left(\frac{R}{r}\right)^n P_n^{(k)}(\cos\theta)[C_{nk}\cos k\varphi + S_{nk}\sin k\varphi]\right\} \tag{1}$$

where r, θ, φ are spherical coordinates of the point in the space. Where in: $x = r\sin\theta\cos\varphi$, $y = r\sin\theta\sin\varphi$, $z = r\cos\theta$, r – is the radius vector, $r = \sqrt{x^2+y^2+z^2}$, φ – is the longitude, θ – is the polar angle; $R = \sqrt[3]{abc}$ – is an average volume radius of the body, $P_n(z)$ is Legendre polynomial determined according to Rodrigues's formula [6]:

$$P_n(z) = \frac{1}{2^n n!}\frac{d^n}{dz^n}(z^2-1)^n,$$

$P_n^{(k)}(z)$ - is the associated Legendre function determined as:

$$P_n^{(k)}(z) = \frac{(1-z^2)^{\frac{k}{2}}}{1}\frac{d^k}{dz^k}P_n(z),$$

J_n, C_{nk}, S_{nk} - are harmonic coefficients determined by the density distribution $\rho(r', \theta', \varphi')$ inside the body.

Laplace's series converges outside some sphere covering the celestial body. Near the celestial body figure, the series is divergent, and the complexity of calculating its total member is increasing.

However, if the coordinate system x, y, z is placed so that the coordinate origin coincides with the center of the body mass, and the coordinate axes coincided with the principal inertia axes, the coefficients $J_1, C_{11}, S_{11}, C_{21}, S_{21}, S_{22} = 0$. Then the first nonzero coefficients would be harmonic coefficients of the second order (Table 1):

$$J_2 = \frac{2I_{zz} - I_{xx} - I_{yy}}{2MR^2}, \tag{2}$$

$$C_{22} = \frac{I_{yy} - I_{xx}}{4MR^2}, \tag{3}$$

where I_{zz}, I_{xx}, I_{yy} - are the main central moments of the triaxial ellipsoid inertia by axes a, b, c determined as follows:

$$I_{xx} = \frac{M}{5}(b^2 + c^2), \quad I_{yy} = \frac{M}{5}(a^2 + c^2), \quad I_{zz} = \frac{M}{5}(a^2 + b^2). \tag{4}$$

The coefficients of the second order have the following mechanical content. They describe the ellipsoid of the body inertia and determine the combination of moments of the body inertia relative to the inertia axes. In the case of self-gravitating satellites, the symmetry of their structure makes it easy to determine the gravitational potential [2]. The external gravitational potential of a homogeneous ellipsoid can be approximately represented as follows:

$$\Phi(r, \theta, \varphi) \approx \frac{GM}{r}\left\{1 - J_2\left(\frac{R}{r}\right)^2 P_2(\cos\theta) + C_{22}\left(\frac{R}{r}\right)^2 P_2^{(2)}(\cos\theta)\cos 2\varphi\right\}. \tag{5}$$

Let's switch to Cartesian coordinates and substitute values (2), (3) and (4). Thus we get [9]:

$$\Phi(r, \theta, \varphi) \approx \frac{GM}{r} \times \\ \times\left\{1 + \frac{(2a^2 - b^2 - c^2)x^2 + (2b^2 - a^2 - c^2)y^2 + (2c^2 - a^2 - b^2)z^2}{10\,r^4}\right\}. \tag{6}$$

Hence we can calculate the external gravitational potential of satellites adopted as homogeneous ellipsoids (Table 2), for example, for Ganymede at the point (2634.4;0;0):

$$\Phi(2634.4; 0; 0) \approx 3.7523 \cdot 10^6 \frac{m^2}{s^2}.$$

For our calculations we selected 18 self-gravitating planets satellites [10]. Obviously, the potential is a scalar function of the force field. Its physical meaning is

work (equals the same) to be performed against the forces of gravity to move the test mass by infinity. From the analysis of Tables 1 and 2 we can conclude that the gravitational potential on the satellite surface increases from the equator to the poles (Fig. 2).

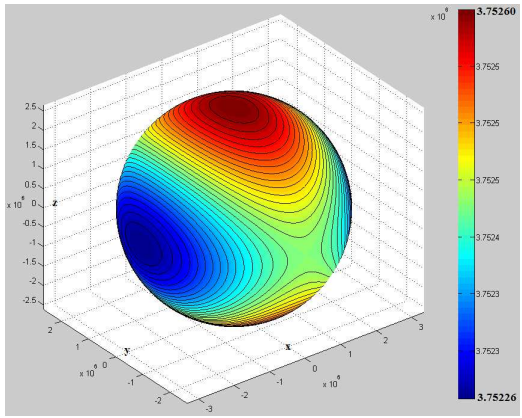

Fig. 2. Gravitational potential on the satellite surface (on the example of Ganymede).

4. DISCUSSIONS AND CONCLUSIONS

1. Gravity field of planet satellites is defined by their shape, mass and internal density distribution. The basic parameters of the gravitational field of planet satellites were determined by the approximation method (provided the homogeneity of their structure and shape of triaxial ellipsoid).

2. The gravitational potential at the surface of satellites depends on the location of latitude of the surface point on the axially symmetric figures, and also on the longitude in the case three-axial satellite figure. Bringing the gravitational field to the Cartesian coordinate system and the combination of the principal inertia axes with the coordinate system axes simplifies the determination of harmonic coefficients and gravitational potential of ellipsoids.

3. Mathematical models of three-dimensional gravity field of satellites as equipotential surfaces were offered.

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Table 1. The coefficients of the gravitational field of planetary satellites

№	Name	Mass, M 10^{20} kg	$A.R.^*$, R 10^3 m	a, 10^3 m	b, 10^3 m	c, 10^3 m	J_2, 10^{-6}	C_{22}, 10^{-6}	J_2/C_{22}
1	Ganymede	1481.90	2634.10	2634.40	2634.10	2633.80	68.33	11.39	5.999
2	Titan	1345.20	2576.00	2578.00	2576.00	2574.00	465.77	77.66	5.997
3	Callisto	1077.00	2410.30	2411.80	2410.30	2408.80	373.36	62.25	5.998
4	Io	893.19	1821.32	1830.00	1818.70	1815.30	1987.86	621.46	3.199
5	Moon	734.80	1737.07	1738.14	1737.10	1735.97	379.91	59.89	6.343
6	Europe	480.20	1560.80	1561.40	1560.80	1560.20	230.64	38.45	5.998
7	Triton	214.00	1353.47	1354.50	1353.40	1352.50	428.48	81.30	5.270
8	Titania	35.27	788.40	790.00	788.40	786.80	1217.24	203.15	5.992
9	Oberon	30.14	761.40	764.00	761.40	758.80	2047.69	342.06	5.986
10	Rhea	23.07	763.90	766.20	763.30	762.20	1335.23	380.05	3.513
11	Iapetus	18.06	731.72	747.40	735.80	712.40	15871.99	1606.71	9.879
12	Charon	15.80	605.97	614.00	606.00	598.00	7904.15	1328.97	5.948
13	Ariel	13.50	578.90	581.10	577.90	577.70	1244.62	553.35	2.249
14	Umbriel	11.75	584.70	586.10	584.70	583.30	1436.06	239.73	5.990
15	Dione	10.96	561.77	564.40	561.30	559.60	2313.39	552.89	4.184
16	Tethys	6.18	531.11	538.40	528.70	526.30	5464.76	1834.75	2.979
17	Enceladus	1.10	252.08	256.60	251.40	248.30	9032.66	2078.55	4.346
18	Miranda	0.71	235.82	240.40	234.20	232.90	7475.09	2645.62	2.826

*Average Radius - A.R.

Table 2. The gravitational potential of planetary satellites

№	Name	Mass, M 10^{20} kg	a, 10^3 m	b, 10^3 m	c, 10^3 m	$\Phi_{x,0,0}$ 10^6 m^2/s^2	$\Phi_{0,y,0}$ 10^6 m^2/s^2	$\Phi_{0,0,z}$ 10^6 m^2/s^2
1	Ganymede	1481.90	2634.40	2634.10	2633.80	3.7523	3.7524	3.7526
2	Titan	1345.20	2578.00	2576.00	2574.00	3.4820	3.4831	3.4842
3	Callisto	1077.00	2411.80	2410.30	2408.80	2.9796	2.9804	2.9811
4	Io	893.19	1830.00	1818.70	1815.30	3.2647	3.2729	3.2753
5	Moon	734.80	1738.14	1737.10	1735.97	2.8208	2.8215	2.8222
6	Europe	480.20	1561.40	1560.80	1560.20	2.0518	2.0521	2.0524
7	Triton	214.00	1354.50	1353.40	1352.50	1.0543	1.0546	1.0549
8	Titania	35.27	790.00	788.40	786.80	0.2982	0.2984	0.2986
9	Oberon	30.14	764.00	761.40	758.80	0.2637	0.2640	0.2644
10	Rhea	23.07	766.20	763.30	762.20	0.2012	0.2015	0.2016
11	Iapetus	18.06	747.40	735.80	712.40	0.1631	0.1642	0.1663
12	Charon	15.80	614.00	606.00	598.00	0.1730	0.1739	0.1748
13	Ariel	13.50	581.10	577.90	577.70	0.1553	0.1556	0.1557
14	Umbriel	11.75	586.10	584.70	583.30	0.1339	0.1340	0.1342
15	Dione	10.96	564.40	561.30	559.60	0.1299	0.1302	0.1303
16	Tethys	6.18	538.40	528.70	526.30	0.0772	0.0778	0.0779
17	Enceladus	1.10	256.60	251.40	248.30	0.0289	0.0291	0.0293
18	Miranda	0.71	240.40	234.20	232.90	0.0199	0.0201	0.0202

ON THE OCCASION OF CENTENARY OF THE BIRTH: SERGEI IVANOVICH SORIN

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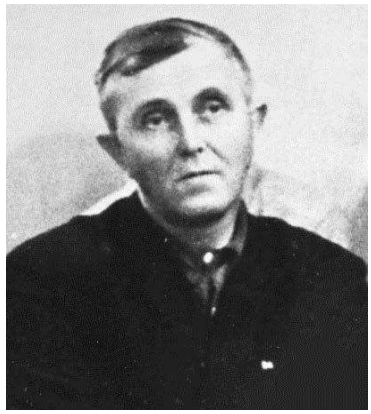

I attended the astronomical circle of the Baku house of pioneers at the very beginning of 1950s, about 2 years (approximately from the end of the 5th and before the beginning of the 8th grade). I cannot remember that there had been much talk on astronomy with my parents or among the close relatives before. Most likely, interest in astronomy has arisen after reading one or two books. Surnames Vorontsov-Velyaminov and even Flamarion were already known to me though I have seen the book of the last much later. However, when I attended the circle, I found with surprise that many of my close friends were interested in astronomy, a stream of questions poured on me, many of which were left without answer for obvious reasons, but nevertheless they began calling me "astronomer" for fun.

During this period Sergei Ivanovich Sorin was deeply involved in the creation of the reflecting optical telescope, according to him, the biggest in the republic. Therefore one of the main tasks of members of the circle was grinding of a glassy ingot with a diameter about 10 inches, similar to a very thick illuminator

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glass. Periodically Sergei Ivanovich checked the results of our work using a small bright lamp for these purposes and examining its image reflected from the future mirror. All of us watched closely this process and listened to Sergei Ivanovich's comments. Then he formulated concrete recommendations to which all should follow, who worked on a mirror. From time to time, if someone joined our circle, Sergei Ivanovich with laughter quoted the statement, apparently, of the specialist, who was very famous in the world in production of telescopes. The sense of this statement was that only hopeless fanatics who do not imagine all complexity of the work can grind an optical mirror of such diameter.

I should mention that it was not the only case when Sergei Ivanovich called into question the statement of even very authoritative specialists. Preparation of the telescope was completed when I already stopped my going to the circle. But, if I remember right, they said that this tool became the property of ASU.

As far as I remember, conversations on scientific subjects, as a rule, arose spontaneously, during informal discussions of the questions asked by members of the circle. Many of these questions were related to past or planned astronomical observations performed in summer, when members of an astronomical circle led by Sergei Ivanovich left to the mountain Pirkuli where the camp was set up and astronomical observations with small telescopes were carried out. As far as I understand, these astronomical observations had played great or even leading role for selecting the location for future Shamakhy observatory. I vividly remember enthusiastic exclamations of the head of department of Physics of the Sun of Pulkovo Observatory, (later its director) Vladimir Alekseevich Krat - "What a granulation!", "No, what a fine granulation!", - when they showed the pictures of the Sun obtained by members of the summer camp. I think the visit of V. A. Krat to our circle was at the end of 1951 when the All-Union conference devoted to the 750th anniversary of Nassir al-Din al-Tusi was held in Baku.

Undoubtedly, Sergei Ivanovich Sorin has played a significant role in the selection of the place for future Shamakhy observatory. In this regard another indirect dispute with acknowledged specialist in astroclimate problems, the member of the Paris Academy Andre-Louis Danjon remains in memory. Sergei Ivanovich repeatedly noted that, according to Danjon, optical observatories must be established only on extensive plateaus. His own experience taught him that good conditions for optical observatory can be implemented also in other places, in particular, similar to mountain Pirkuli slopes. Without being limited only with demonstration of good photos of the Sun and sky of stars, Sergei Ivanovich tried even "on fingers" to present how and due to what on a slope of the mountain the turbulent streams which harmfully affect the quality of images of astronomical objects, can be substantially compensated.

Sergei Ivanovich was well-known among members of All-Union Astrogeodetic Society. In 1950 or 1951 a letter from members of the astronomical circle in Kuibyshev (now Samara) which was directed by other enthusiast, Sergei Ivanovich's friend came to our circle. This episode remained in my memory as the management of the House of pioneers decided that discussion of our collective answer to this letter should be shown in a newsreel (in one of newsreels which were shown those years at movie theaters before showing the main films). The team from the film studio arrived and in two (if not more) hours members of the circle of our group felt as movie stars. Approximately in 2015 a group of young fans of astronomy from Samara together with two-three parents and two female-directors of children's groups of fans of astronomy came to us to Pushchino Radio Astronomy Observatory on an excursion. I won't deny, it was very pleasant to me to learn that one of these directors (who was some years older than me) remembered a little later contacts of Samara and Baku children's astronomical circles. She exactly remembered Sergei Ivanovich Sorin (in the mid-1950s she was already a student, but continued to participate in the work of an astronomical circle). One more episode related to filming belongs to the second half of the 1960s. I already worked in Pushchino Radio Astronomy Observatory, when Baku documentary film makers called me and proposed to meet to share the memories about the astronomical circle of the Baku house of pioneers. Shootings were made near the viewing platform on Lenin Mountains in Moscow. I tried to remember the most interesting moments, and all of them, of course, were connected with Sergei Ivanovich. I was finally interrupted, asked the following: "Why are talking only about Sergei Ivanovich? Weren't there other people?". It seemed to me that they wanted to hear some other name, but with all the will I couldn't remember anyone else.

My last meeting with Sergei Ivanovich was approximately at the end of the 1960s or at the beginning of the 1970s. During one of visits to Baku I went to the new room of the astronomical circle (if I am not mistaken, on Samad Vurgun Street or parallel to it). Sergei Ivanovich asked me to tell children about radio astronomy and our observatory. As far as I remember, there were several adults among listeners maybe parents of children. Never see Sergei Ivanovich again for what I really regret. Looking back to the past, I can tell firmly - Sergei Ivanovich Sorin's influence on formation of my relation to work, to people and life, in general, is very great. I don't think that he specially set himself for this purpose, when communicated with the children. But his relation to favorite work in which he put all his life, was involuntarily transferred to the children around him. And all of us involuntarily felt his assessment of the events taking place around, and, in general, his idea of what is good and what is bad.